8th World Congress on Structural and Multidisciplinary Optimization
June 1 - 5, 2009, Lisbon, Portugal

Enhanced Probabilistic Analytical Target Cascading with Application to Multiscale Design

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Abstract
Probabilistic Analytical Target Cascading (PATC) is an approach for multilevel multidisciplinary design optimization under uncertainty. In the existing PATC approach, only the mean and variance of each individual interrelated response and linking variable are matched in a multilevel hierarchy. However, due to the existence of random linking variables or parameters, the interrelated responses from lower-level subsystems are statistically correlated and have a direct influence on the statistical performance of an upper-level subsystem. The ignorance of response correlation introduces difficulties in finding optimal solutions especially when the covariance of interrelated responses has a significant impact. In this paper, an enhanced PATC (EPATC) approach is proposed to improve the performance of PATC by considering the correlations in optimization cycles. With the EPATC approach, in addition to matching the first two statistical moments, the covariance between the interrelated responses is also considered by applying a modified updating strategy for estimating the statistical performance of an upper-level subsystem. A mathematical example and a multiscale design problem are used to demonstrate the effectiveness and efficiency of the proposed EPATC approach. The results using the Probabilistic All-In-One (PAIO) method are used as references to verify the accuracy of the EPATC approach. It is observed that the effectiveness of EPATC highly depends on the impact strength of the covariance on optimal solutions. Our study shows that the EPATC approach outperforms the original PATC by providing more accurate optimal solutions; the multilevel optimization that allows distributed design activities is highly applicable to multiscale design problems.

Keywords: Probabilistic analytical target cascading, Multilevel optimization, Uncertainty, Correlated response, Multiscale design

1. Introduction

Complex systems design often involves a large number of design variables and information couplings among subsystems. The traditional All-In-One (AIO) optimization method may be impractical because of the high cost associated with the computation of a fully coupled system. Multidisciplinary design optimization (MDO) [1-8] methods have been developed to relieve the burden by decomposing a system into several manageable subsystems either in a non-hierarchical or hierarchical (multi-level) fashion. Among them, the Analytical Target Cascading (ATC) method [4-8] is a hierarchical MDO approach that employs multilevel optimization formulations. On the other hand, it is widely recognized that uncertainty universally exists in engineering systems and often causes unexpected quality loss or catastrophic failure [9,10]. The MDO methods have been extended to MDO under uncertainty (MDO-U) for designing reliable and robust systems, including Multidisciplinary Robust Design Optimization (MRDO) [11,12] and Reliability-Based Multidisciplinary Design Optimization (RBMDO)[13,14]. Most of these MDO-U methods are developed for non-hierarchical systems rather than for multilevel systems. The ATC method was extended to a probabilistic formulation first by Kokkolaras et al. [15], in which only the mean values of the interrelated responses and linking variables were matched between neighboring levels. A Probabilistic Analytical Target Cascading (PATC) formulation was later developed by Liu et al. [16] to match not only the mean but also the variance values of the interrelated responses and linking variables. It was demonstrated that PATC can obtain almost the same result as that of Probabilistic AIO (PAIO) when the first two moments can sufficiently describe the probabilistic characteristics of random variables.

One major characteristic of multilevel hierarchical systems is that the outputs of lower-level subsystems act as inputs to upper-level subsystems, and thus named *interrelated responses*. Often times these interrelated responses

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are correlated to each other with respect to the same uncertainty source. Figure 1 shows two typical cases of bi-level systems with interrelated responses where covariance may exist. In Figure 1(a), due to the common uncertainty source in the linking variable Y, the two responses R_{11} and R_{12} from their corresponding subsystems are statistically correlated, as named *correlated responses* in this paper. In Figure 1(b), the two responses from a single subsystem share the same two random inputs and therefore are statistically correlated. Such a case (Figure 1(b)) can be transformed to the first case (Figure 1(a)) by splitting the interrelated responses R_{11} and R_{12} into two individual subsystems sharing two linking variables (X_{11} and X_{12}). Ignoring the covariance in the existing PATC implementation results in an insufficient statistical representation of the correlated responses, and thus may yield an inaccurate estimation of statistical performance at the upper level. In hierarchical MDO methods, design targets can be set for the mean and variance of interrelated responses and cascaded down to each *individual* lower-level subsystem designs. However, it is not straightforward to cascade targets for the performance covariance to lower-level subsystem designs since the covariance is assessed based on the interrelated responses from *multiple* subsystems.

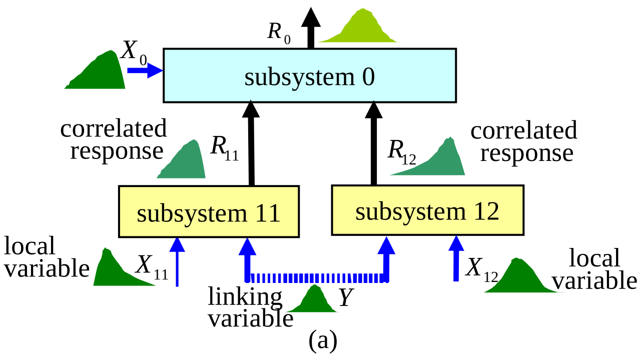

Figure 1 Two types of bi-level system with correlated responses

In this paper, an enhanced PATC (EPATC) approach is proposed to take into account the covariance of the interrelated responses in the hierarchical optimization process. In addition to matching the mean and variance of the interrelated responses and linking variables, the covariance between the correlated responses is incorporated into the optimization formulation by introducing a modified updating strategy for estimating the upper-level statistical performance. Together with the mean and variance values of interrelated responses in the current optimization cycle, correlation coefficients from the previous cycle are used to update the probabilistic characteristics of the interrelated responses, and consequently update the upper-level statistical performance. The rest of the paper is organized as follows. In Section 2, the PATC formulation and its limitations are first reviewed and the proposed EPATC approach is then presented. In Section 3, a mathematical example is tested to demonstrate the efficiency and accuracy of the proposed approach. In Section 4, the EPATC approach is applied to a multiscale bracket design problem that integrates both material and product design. Section 5 provides the conclusion.

2. Enhanced PATC (EPATC) Approach

2.1 Technical Background of PATC

Figure 2 shows the design information flow of subsystem j at level i ($\mathbf{O}_{\text{ij}}$) in the original PATC approach. For a subsystem at certain level, its neighboring lower-level subsystems are called its children, while the neighboring upper-level subsystems are called parents. In Figure 2, $\mathbf{O}_{\text{ij}}$ has n_{ij} children in the lower levels with R_{ij} and Y_{ij} are its random response and linking variable vectors, respectively. Rij are evaluated using analysis or simulation models $\mathbf{R}_{ij} = \mathbf{f}_{ij}\left(\mathbf{R}_{(i+1)1},\ldots,\mathbf{R}_{(i+1)n_{ij}},\mathbf{X}_{ij},\mathbf{Y}_{ij}\right)$, which are functions of local variables $\mathbf{X}_{ij}$, linking variables $\mathbf{Y}_{ij}$ and children responses $\mathbf{R}_{(i+1)k}$ (k=1,…,n_{ij}). The corresponding PATC formulation for subsystem $\mathbf{O}_{\text{ij}}$ is shown in Eq.(1).

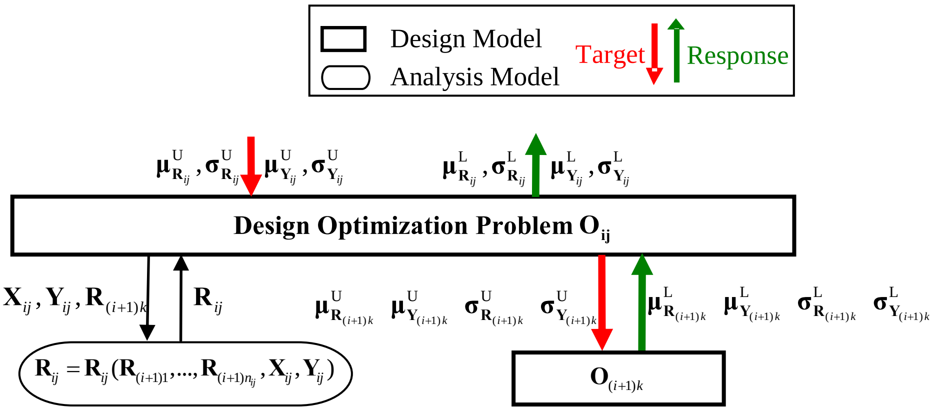

Figure 2: Information flow in PATC [16]

Targets for the mean and standard deviation of $\mathbf{R}_{ij}$ and $\mathbf{Y}_{ij}$ are assigned by the parent as $\boldsymbol{\mu}^{U}_{R_{ij}}, \boldsymbol{\sigma}^{U}_{R_{ij}}$ and $\boldsymbol{\mu}^{U}_{Y_{ij}}, \boldsymbol{\sigma}^{U}_{Y_{ij}}$ respectively. The outputs of the subsystem $\mathbf{O}_{ij}$ optimization problem are the achievable mean and standard deviation of $\mathbf{R}_{ij}$ and $\mathbf{Y}_{ij}$, and passed up to its parent as $\boldsymbol{\mu}^{L}_{R_{ij}}, \boldsymbol{\sigma}^{L}_{R_{ij}}$ and $\boldsymbol{\mu}^{L}_{Y_{ij}}, \boldsymbol{\sigma}^{L}_{Y_{ij}}$. Similarly, achievable values of children responses and linking variables are passed up to $\mathbf{O}_{ij}$ as $\boldsymbol{\mu}^{L}_{R_{(i+1)k}}, \boldsymbol{\sigma}^{L}_{R_{(i+1)k}}$ and $\boldsymbol{\mu}^{L}_{Y_{(i+1)k}}, \boldsymbol{\sigma}^{L}_{Y_{(i+1)k}}$, taking into account the design consistency which are formulated as the first four constraints in Eq.(1). The optimization problem for subsystem $\mathbf{O}_{ij}$ is solved to find the optimum values for its local design variables $\mathbf{X}_{ij}$, linking variables $\mathbf{Y}_{ij}$ and the target for its children's responses [$\boldsymbol{\mu}^{U}_{R_{(i+1)k}}, \boldsymbol{\sigma}^{U}_{R_{(i+1)k}}$] and linking variables [$\boldsymbol{\mu}^{U}_{Y_{(i+1)k}}, \boldsymbol{\sigma}^{U}_{Y_{(i+1)k}}$], to minimize the discrepancy between parent targets $\boldsymbol{\mu}^{U}_{R_{ij}}, \boldsymbol{\sigma}^{U}_{R_{ij}}, \boldsymbol{\mu}^{U}_{Y_{ij}}, \boldsymbol{\sigma}^{U}_{Y_{ij}}$ and $\boldsymbol{\mu}_{R_{ij}}, \boldsymbol{\sigma}_{R_{ij}}, \boldsymbol{\mu}_{Y_{ij}}, \boldsymbol{\sigma}_{Y_{ij}}$ respectively, and to satisfy the consistency constraints as well as the probabilistic constraints. The completion of all the optimizations of the subsystems in the hierarchy is considered as one optimization *cycle*. With PATC, targets (mean and variance) are set and cascaded down by the upper-level subsystems (parents) to the responses and linking variables of the lower levels (children), while the lower-level subsystems try to match these targets and feeding back their achievable values. Such process of "target cascading – response feedback" will continue level by level, then cycle after cycle till the convergence criterion is achieved.

$$
\begin{aligned}
&\text{Given} && \boldsymbol{\mu}^{U}_{R_{ij}}, \boldsymbol{\sigma}^{U}_{R_{ij}}, \boldsymbol{\mu}^{U}_{Y_{ij}}, \boldsymbol{\sigma}^{U}_{Y_{ij}}, \boldsymbol{\mu}^{L}_{R_{(i+1)k}}, \boldsymbol{\sigma}^{L}_{R_{(i+1)k}}, \boldsymbol{\mu}^{L}_{Y_{(i+1)k}}, \boldsymbol{\sigma}^{L}_{Y_{(i+1)k}}, \quad k = 1, \ldots, n_{ij} \\
&\text{find} && \boldsymbol{\mu}_{R_{(i+1)k}}, \boldsymbol{\sigma}_{R_{(i+1)k}}, \mathbf{X}_{ij}, \boldsymbol{\mu}_{Y_{ij}}, \boldsymbol{\sigma}_{Y_{ij}}, \boldsymbol{\mu}_{Y_{(i+1)k}}, \boldsymbol{\sigma}_{Y_{(i+1)k}}, \varepsilon^{\boldsymbol{\mu}_R}_{ij}, \varepsilon^{\boldsymbol{\sigma}_R}_{ij}, \varepsilon^{\boldsymbol{\mu}_Y}_{ij}, \varepsilon^{\boldsymbol{\sigma}_Y}_{ij}, \quad k = 1, \ldots, n_{ij} \\
&\text{min} && \left\| \boldsymbol{\mu}_{R_{ij}} - \boldsymbol{\mu}^{U}_{R_{ij}} \right\| + \left\| \boldsymbol{\sigma}_{R_{ij}} - \boldsymbol{\sigma}^{U}_{R_{ij}} \right\| + \left\| \boldsymbol{\mu}_{Y_{ij}} - \boldsymbol{\mu}^{U}_{Y_{ij}} \right\| + \left\| \boldsymbol{\sigma}_{Y_{ij}} - \boldsymbol{\sigma}^{U}_{Y_{ij}} \right\| + \varepsilon^{\boldsymbol{\mu}_R}_{ij} + \varepsilon^{\boldsymbol{\sigma}_R}_{ij} + \varepsilon^{\boldsymbol{\mu}_Y}_{ij} + \varepsilon^{\boldsymbol{\sigma}_Y}_{ij} \\
&\text{s.t.} && \sum_{k=1}^{n_{ij}} \left\| \boldsymbol{\mu}_{R_{(i+1)k}} - \boldsymbol{\mu}^{L}_{R_{(i+1)k}} \right\| \le \varepsilon^{\boldsymbol{\mu}_R}_{ij}, \quad \sum_{k=1}^{n_{ij}} \left\| \boldsymbol{\sigma}_{R_{(i+1)k}} - \boldsymbol{\sigma}^{L}_{R_{(i+1)k}} \right\| \le \varepsilon^{\boldsymbol{\sigma}_R}_{ij} \\
& && \sum_{k=1}^{n_{ij}} \left\| \boldsymbol{\mu}_{Y_{(i+1)k}} - \boldsymbol{\mu}^{L}_{Y_{(i+1)k}} \right\| \le \varepsilon^{\boldsymbol{\mu}_Y}_{ij}, \quad \sum_{k=1}^{n_{ij}} \left\| \boldsymbol{\sigma}_{Y_{(i+1)k}} - \boldsymbol{\sigma}^{L}_{Y_{(i+1)k}} \right\| \le \varepsilon^{\boldsymbol{\sigma}_Y}_{ij} \\
& && \Pr\left[g_{ij,m}\left(\mathbf{R}_{ij}, \mathbf{X}_{ij}, \mathbf{Y}_{ij} \right) \le 0 \right] \ge \alpha_{ij,m}, \quad m = 1, \ldots, M \\
&\text{where} && \mathbf{R}_{ij} = \mathbf{f}_{ij}\left(\mathbf{R}_{(i+1)1}, \ldots, \mathbf{R}_{(i+1)n_{ij}}, \mathbf{X}_{ij}, \mathbf{Y}_{ij} \right)
\end{aligned} \tag{1}
$$

In a multilevel (e.g. bi-level) hierarchy (see Figure 1(a)), uncertainties associated with the lower-level inputs X_{11}, X_{12} and Y are propagated to the upper-level interrelated subsystem responses R_{11}, R_{12} and further to the top-level subsystem performance R_0. Uncertainty will exist at any level in the hierarchy and it is very important to propagate the uncertainty across different levels accurately. The distributions of R_{11} and R_{12} are correlated as a result of sharing a common uncertainty source associated with the linking variable Y. Such correlation cannot be ignored for an accurate assessment of the top-level statistical performance R_0 in the hierarchical system. In the original PATC formulation and solution procedure, the uncertainties of R_{11}, R_{12} are propagated to the upper-level responses only in terms of mean and variance values, without considering the covariance between the interrelated responses. When the statistical performance measures of the upper-level responses are inaccurate, it consequently results in an inferior or even inconsistent optimal solution at the end of the iterative optimization process.

2.2 Description of EPATC

To overcome the limitation of PATC in dealing with correlated responses, the enhanced probabilistic analytical target cascading (EPATC) approach is proposed in this paper. With EPATC, these correlated responses are assumed as multi-variant normal distributions rather than uncorrelated normal ones in the original PATC. Based on the PATC formulation as shown in Eq. (1), a modified strategy for updating the statistical performances at the upper level of a hierarchy is developed and therefore the uncertainties are propagated more accurately across the hierarchy. A flowchart of the EPATC with a top-down strategy for a demonstrative bi-level design problem is illustrated in Figure 3. X_opt denotes the optimal design variables at all the levels including local design variables and linking design variables. The dashed box represents the process of computing the correlation coefficients of the correlated responses based on the optimal solution X_opt obtained in cycle *S*, which can be described as three steps below:

Step 1. Distribute the current optimal solution X_opt to each subsystem at all levels in the hierarchy;

Step 2. For each subsystem, based on the optimal values obtained in step 1 and the distribution parameters, calculate its output response by Monte Carlo method [17];

Step 3. Calculate the correlation coefficients (cov) between the correlated responses.

As mentioned in Introduction, the covariance cannot be obtained as the "target" performance in the current optimization cycle. Therefore, in the next cycle S+1 (S>1), these correlation coefficients obtained from the previous cycle S together with the mean and variance of the interrelated responses in the current cycle S+1 will be used to estimate the probabilistic characteristics (mean and variance) of the upper-level responses. Different from the original PATC in which the dash box in Figure 4 is not introduced, the covariance is estimated and then taken into account in EPATC. Such a treatment yields more accurate statistical assessments in the upper levels when the impact of covariance cannot be ignored.

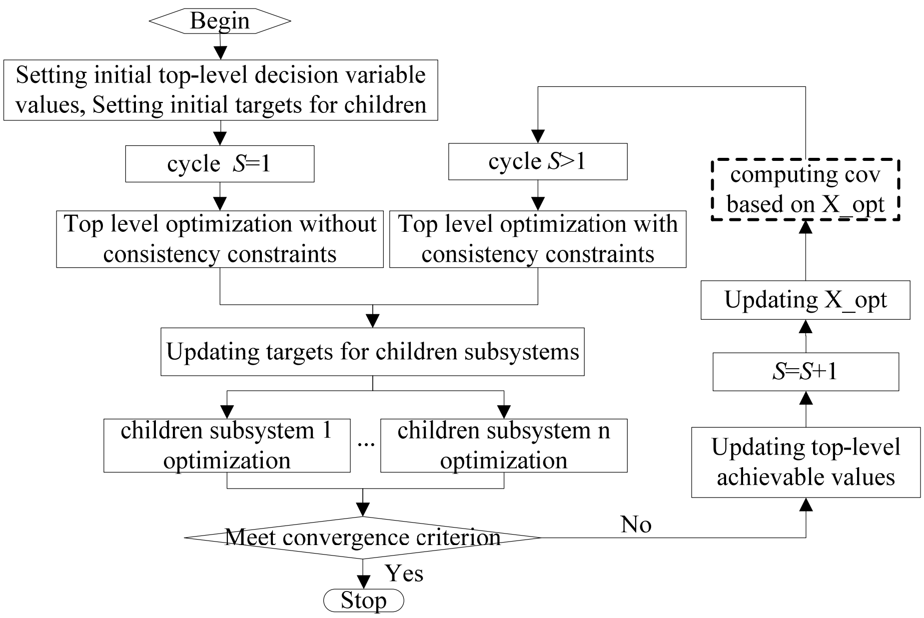

Figure 3: Flowchart of EPATC [18]

3. Case Study

In this section, a mathematical example is used to demonstrate the benefits of the proposed EPATC approach. The deterministic all-in-one (AIO) optimization problem is formulated in Eq. (2) with four design variables. Both the optimal design variables X of EPATC and PATC are plugged into the PAIO formulation to obtain the confirmed objective f and constraint g function values. The optimal solutions (X, confirmed f and confirmed g) are verified by comparing to the ones using the PAIO method. In order to demonstrate the impact of the magnitude of covariance on the optimal solutions, different sets of standard deviation values (STD) of random design variables are tested. To eliminate the influence of different optimization process settings such as initial start points, stopping criterion, and weights in PATC formulation, all the settings in both the approaches (PATC and EPATC) are kept the same. The Monte Carlo method is used to calculate all the probabilistic characteristics of the objective and constraint functions. All probabilistic constraints are simplified into the moment-matching formulation, i.e., $\mu_g + k\sigma_g \le 0$. The required reliability level α is 99.865% (k=3) for all probabilistic constraints. Multiple starting points are tested to find global optimal solutions. This mathematical example is formulated as a bi-level design structure by EPATC shown in Figure 4.

$$
\begin{aligned}
\min \quad & -x_1 R_{11} R_{12} \\
\text{s.t.} \quad & 0 \le x_1 + 2R_{11} + 2R_{12} - 27 \le 0 \\
& x_2^2 + x_4^2 - 2.5 \le 0 \\
& x_3^2 + x_4^2 - 1.25 \le 0 \\
\text{where } & R_{11} = (x_2 + x_4)^2, \\
& R_{12} = (x_3 + x_4)^2 \\
& x_1, x_2\ x_3, x_4 \ge 0
\end{aligned}
\tag{2}
$$

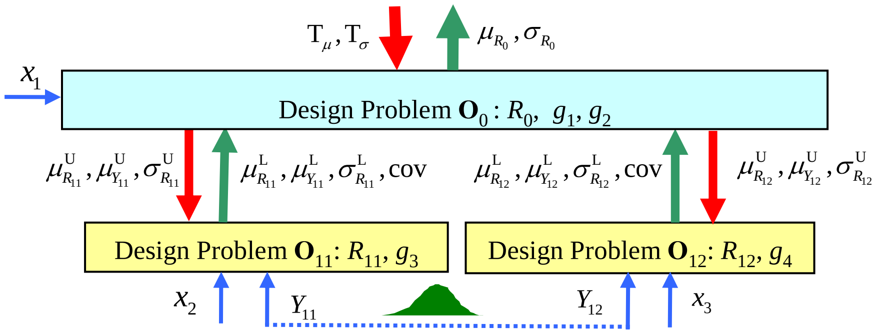

Figure 4: Information flow in EPATC of the mathematical example

There are two subsystems at the bottom level and each has one response (R_{11} and R_{12} respectively) passed to the top level. x_1, x_2 and x_3 are the local design variables that belong to each subsystem across two levels. The linking variable Y, which is denoted as x_4 in the AIO formulation, is considered as the only random design variable subject to a normal distribution. The correlation coefficient between R_{11} and R_{12} from the previous cycle is denoted as cov. The EPATC formulation is provided in Eqs.(3)-(5) in which $\mathbf{O}_0$ is the top-level optimization problem and $\mathbf{O}_{11}$,$\mathbf{O}_{12}$ are the two bottom-level optimization problems.

$$
\begin{aligned}
\mathbf{O}_0: & \text{Given } \mathrm{T}^\mu, \mathrm{T}^\sigma, \mu^L_{R_{11}}, \sigma^L_{R_{11}}, \mu^L_{R_{12}}, \sigma^L_{R_{12}}, \mu^L_{Y_{11}}, \mu^L_{Y_{12}}, \\
\text{find} \quad & x_1, \mu_{R_{11}}, \sigma_{R_{11}}, \mu_{R_{12}}, \sigma_{R_{12}}, \mu_Y, \varepsilon^{\mu_R}, \varepsilon^{\sigma_R}, \varepsilon^{\mu_Y} \\
\min \quad & (\mathrm{T}^\mu - \mu_{R_0})^2 + (\mathrm{T}^\sigma - \sigma_{R_0})^2 + w_1\varepsilon^{\mu_R} + w_2\varepsilon^{\sigma_R} + w_3\varepsilon^{\mu_Y} \\
\text{s.t.} \quad & (\mu_{R_{11}} - \mu^L_{R_{11}})^2 + (\mu_{R_{12}} - \mu^L_{R_{12}})^2 \le \varepsilon^{\mu_R} \\
& (\sigma_{R_{11}} - \sigma^L_{R_{11}})^2 + (\sigma_{R_{12}} - \sigma^L_{R_{12}})^2 \le \varepsilon^{\sigma_R} \\
& (\mu_Y - \mu^L_{Y_{11}})^2 + (\mu_Y - \mu^L_{Y_{12}})^2 \le \varepsilon^{\mu_Y} \\
& \mu_{g_1} + 3\sigma_{g_1} \le 0, \mu_{g_2} + 3\sigma_{g_2} \le 0 \\
\text{where } & R_0 = -x_1 R_{11} R_{12} \\
& g_1 = (x_1 + 2R_{11} + 2R_{12})/27 - 1 \le 0 \\
& g_2 = -(x_1 + 2R_{11} + 2R_{12}) \le 0
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
&\mathbf{O}_{11}\text{:Given } \mu_{R_{11}}^{\mathrm{U}}, \sigma_{R_{11}}^{\mathrm{U}}, \mu_{Y}^{\mathrm{U}}, \sigma_{Y_{11}} \\
&\text{find} \quad x_2, \mu_{Y_{11}} \geq 0 \\
&\text{min} \quad (\mu_{R_{11}} - \mu_{R_{11}}^{\mathrm{U}})^2 + (\sigma_{R_{11}} - \sigma_{R_{11}}^{\mathrm{U}})^2 + (\mu_{Y_{11}} - \mu_{Y}^{\mathrm{U}})^2 \\
&\text{s.t.} \quad \mu_{g_3} + 3\sigma_{g_3} \leq 0 \\
&\text{where } R_{11} = (x_2 + \mu_{Y_{11}})^2, g_3 = (x_2^2 + \mu_{Y_{11}}^2)/2.5 - 1 \leq 0
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
&\mathbf{O}_{12}\text{:Given } \mu_{R_{12}}^{\mathrm{U}}, \sigma_{R_{12}}^{\mathrm{U}}, \mu_{Y}^{\mathrm{U}}, \sigma_{Y_{12}} \\
&\text{find} \quad x_3, \mu_{Y_{12}} \geq 0 \\
&\text{min} \quad (\mu_{R_{12}} - \mu_{R_{12}}^{\mathrm{U}})^2 + (\sigma_{R_{12}} - \sigma_{R_{12}}^{\mathrm{U}})^2 + (\mu_{Y_{12}} - \mu_{Y}^{\mathrm{U}})^2 \\
&\text{s.t.} \quad \mu_{g_4} + 3\sigma_{g_4} \leq 0 \\
&\text{where } R_{12} = (x_3 + \mu_{Y_{12}})^2, g_4 = (x_3^2 + \mu_{Y_{12}}^2)/1.25 - 1 \leq 0
\end{aligned}
\tag{5}
$$

Three different STD values of the random variable Y (STD = 0.01, 0.05, 0.1) are tested. Optimal solutions of the design variables, the objective function and constraint values using EPATC, PATC and PAIO are shown in Table 1 (STD= 0.1). It is observed that compared to PATC, both the optimal design variables and the objective function using EPATC are much closer to the ones using PAIO. Although PATC provides the best (minimum) objective function value among the three methods, the optimal solution of PATC is not feasible since one of the constraints are not satisfied (g_1=0.037). The errors of the design solutions for the three different STD values are also displayed in Table 2. It is found that for all the three STD values, the proposed EPATC approach always performs better than PATC. The larger the STD value is, a strong correlation between the interrelated responses exists and hence the more significant improvement EPATC can achieve compared to PATC. The consideration of covariance in EPATC provides an accurate estimation of the upper-level statistical response and consequently results in an accurate optimal solution of a multilevel design problem.

Table 1: Comparison of optimal solutions (STD=0.1)

	EPATC	PATC	PAIO
X	12.1988 1.3571 0.7708 0.5504	13.2830 1.3421 0.7503 0.5692	12.2076 1.3552 0.7658 0.5604
Relative error (X)	0.12%	8.73%	---
Objective f	5177.7	4249.8	---
Confirmed f	5154.2	4300.3	4986.0
Relative error (f)	+3.37%	-13.75%	---
Constraint g_1	0	0	---
Confirmed g_1	-0.0042	0.037	0

Table 2 : Comparison of optimal solution between EPATC and PATC

	Relative error of X			Relative error of f		
	STD=0.1	STD=0.05	STD=0.01	STD=0.1	STD=0.05	STD=0.01
EPATC	1.68%	2.13%	3.05%	0.62%	7.25%	5.67%
PATC	10.25%	3.96%	3.20%	17.32%	17.76%	12.50%

4. Application of EPATC to Multiscale Bracket Design Problem

Multi-scale design is an emerging research topic that is built upon multiscale simulations to design systems at different scales (length and time) for achieving the required system performances [19]. Recent years have seen work that views multiscale design as a multidisciplinary design activity where design decisions are made by each individual discipline (e.g., material design, product design, and manufacturing process design) with a common objective of achieving the desired product performance [20]. Due to the hierarchical structure of scale decomposition in multiscale systems, there is a potential to exploit the existing hierarchical multidisciplinary design optimization techniques, such as the multilevel optimization methods for making design decisions at various scales. Although PATC has been applied to various multilevel optimization problems in vehicle and aircraft systems design applications, there does not exist any application of PATC to multiscale integrated material and product design. In this section, we illustrate the use of EPATC for multiscale design. The effectiveness of the approach is verified by solving the same problem using the PAIO formulation.

4.1 Problem Description

The multiscale design problem is considered as a multilevel multidisciplinary design problem to design optimal material microstructure and product geometry that yields the minimum volume of material, subject to the stress constraint. Figure 5 illustrates the framework of the multiscale problem as well as the information flow in the bi-level EPATC formulation. The multiscale bracket problem contains one product model at Scale 1 and one material model at Scale 2. At Scale 1, the left vertical surface of the bracket structure is fixed on the wall and the displacement boundary condition is applied to the upper surface of the bracket. There are three product design variables (C_X, C_Y, R) that represent the location and radius of the hole in the bracket. The finite element modeling and analysis is implemented in an ABAQUS© environment to predict the maximum stress, which is expressed in terms of C_X, C_Y, R, *k* and *n* within the bracket when the boundary conditions are fully applied. The strength index (*k*) and strain hardening index (n) are material design parameters from the power model to represent the material constitutive property. At Scale 2, a Representative Volume Element (RVE) material model [21] is employed to construct the microstructure-constitutive property relation of an aluminum alloy material. The aluminum alloy material contains micro silicon particles uniformly distributed in the aluminum matrix. Silicon Particle Volume Fraction (PVF) and Particle Density (N) which quantitatively characterize the material microstructure are introduced as two material design variables. A power model is employed to fit the strain-stress curve from RVE simulations. The fitting process follows the one introduced in Ref. [22] although only the uniform configuration is considered. *k* and *n* act as two interrelated responses passed up to Scale 1. Due to the high computational cost of RVE simulations, the Kriging metamodels [23] are constructed for material property responses (*k* and *n*) as functions of microstructure material design variables PVF and N.

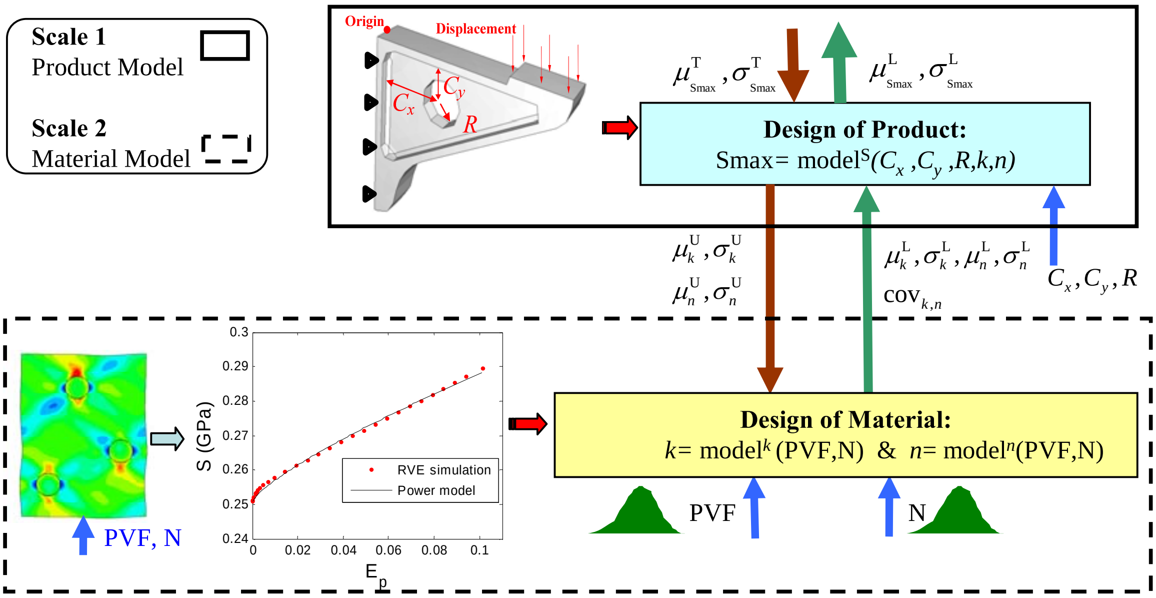

Figure 5: Framework and information flow in EPATC of the multi-scale bracket problem

The design objective is to minimize the material volume used in the bracket product, which is equivalent to maximizing the radius of the hole. The maximum stress occurred in the bracket should be less than the critical stress ($Smax_C$).Additional geometry constraints ($g_2 - g_4$) are applied to ensure the hole within the bracket external contour. The deterministic AIO (all-in-one) optimization formulation of this problem is shown in Eq. (6).

$$
\begin{aligned}
\min \quad & -x_3^2 \\
\text{s.t.} \quad & g_1 = \text{Smax}/\text{Smax}_\text{C} - 1 \le 0 \\
& g_2 = (x_2 + x_3)/20 + 1 \le 0 \\
& g_3 = 1 - (x_1 - x_3)/20 \le 0 \\
& g_4 = 0.5387x_1 - x_2 + 1.1359x_3 \le 0\,, \\
& k = \text{model}^k(x_4, x_5) \\
& n = \text{model}^n(x_4, x_5) \\
& \text{Smax} = \text{model}^\text{S}(x_1, x_2, x_3, k, n) \\
& lb \le X \le ub
\end{aligned} \tag{6}
$$

where $X = \{x_1, x_2, x_3, x_4, x_5\} = \{C_X, C_Y, R, PVF, N\}$, model^k and model^n are the Kriging metamodels for the two interrelated responses *k* and *n*; model^S stands for the Kriging metamodel of the maximum stress; g_1 is the maximum stress constraint. Due to the random nature of material, the microstructure design variables PVF and N are considered as random design variables in this problem. It is found that the two interrelated responses (*k* and *n*) from model Scale 2 are highly correlated. The proposed EPATC approach is applied to the multiscale design problem formulated in Eq.s (7)-(8). Following the target cascading fashion, targets of the desired material design parameters *k* and *n* are determined at Scale 1 (Eq. (7)) and assigned to the Scale 2 design problem (Eq. (8)). The Scale 2 design optimization is carried out to match the targets. When applying the EPATC approach, the obtained covariance of the material responses *k* and *n* is considered in Scale 1 model analysis to ensure a more accurate evaluation of Smax in the next cycle.

Top level (Scale 1):

$$
\begin{aligned}
\min \quad & x_3^2 + w_1\varepsilon^\mu + w_2\varepsilon^\sigma \\
s.t. \quad & P(g_1 \le 0) \ge \alpha \\
& g_1 = \text{Smax}/\text{Smax}_\text{C} - 1 \\
& g_2 = (x_2 + x_3)/20 + 1 \le 0 \\
& g_3 = 1 - (x_1 - x_3)/20 \le 0 \\
& g_4 = 0.5387x_1 - x_2 + 1.1359x_3 \le 0 \\
& (\mu_k - \mu_k^\text{L})^2 + (\mu_n - \mu_n^\text{L})^2 \le \varepsilon^\mu \\
& (\sigma_k - \sigma_k^\text{L})^2 + (\sigma_n - \sigma_n^\text{L})^2 \le \varepsilon^\sigma \\
& \text{Smax} = \text{model}^\text{S}(x_1, x_2, x_3, k, n) \\
& lb \le X \le ub, X = (x_1, x_2, x_3, \mu_k, \sigma_k, \mu_n, \sigma_n, \varepsilon^\mu, \varepsilon^\sigma)
\end{aligned} \tag{7}
$$

Bottom level (Scale 2):

$$
\begin{aligned}
\min \quad & (\mu_k - \mu_k^\text{U})^2 + (\mu_n - \mu_n^\text{U})^2 + (\sigma_k - \sigma_k^\text{U})^2 + (\sigma_n - \sigma_n^\text{U})^2 \\
& k = \text{model}^k(x_1, x_2) \\
& n = \text{model}^n(x_1, x_2) \\
& lb \le X \le ub \\
& X = (x_1, x_2) = (PVF, N)
\end{aligned} \tag{8}
$$

4.2 Optimization Results

Three different sets of STD values (V_1=[0.0067 0.30], V_2=[0.008 0.40], V_3=[0.009 0.45]) for the two random material design variables (PVF and N) are tested. The results using the PAIO method are used as reference to verify the effectiveness of the proposed EPATC approach. Table 3 lists the optimal design variables of EPATC and PAIO, respectively. It is found that the results of EPATC are almost identical with the ones using PAIO for all the STD values with the maximum relative error as 0.17%. The confirmed objective *f* values of PATC are calculated by plugging the optimal design variables into the PAIO formulation. The mean and standard deviation values of the two interrelated responses (*k* and *n*) at the optimal design values and the confirmed *f* are displayed in Table 4. The relative error of the objective function value does not exceed 0.25% compared to the one using PAIO.

Compared to the PAIO method that treats all models at different scale levels as an integrated system, the EPATC follows the hierarchical decomposition strategy in which a complex system is divided into subsystems at different levels, and each subsystem design problem is solved in a target cascading iterative fashion. Such a method maintains the design autonomy of each subsystem at different levels. By considering the covariance between interrelated responses, the EPATC method propagates uncertainties across the multi-levels provides accurate statistical estimations of the responses during optimization. Through the application problem, the EPATC approach demonstrates great advantages and a high applicability for multiscale design problems.

Table 3: Optimal design variables for different STD values

STD	EPATC					PAIO					ΔX / X
V_1	65.4009	-65.0677	45.0354	0.0516	4.0000	65.2398	-65.0912	45.0912	0.0515	4.0000	0.17%
V_2	66.2161	-64.8449	44.8449	0.0540	4.2000	66.2219	-64.8617	44.8417	0.0540	4.2000	0.013%
V_3	68.3668	-64.3025	44.3025	0.0570	4.3500	68.4860	-64.2724	44.2724	0.0570	4.3500	0.12%

Table 4: Objective and interrelated responses for different STD values

STD		V_1	V_2	V_3
Confirmed f	EPATC	2028.2	2011.1	1962.8
	PAIO	2033.2	2010.8	1960.0
Δf / f	---	0.25%	0.0149%	0.140%
μ_k	EPATC	0.2544	0.2533	0.2510
	PAIO	0.2545	0.2533	0.2510
σ_k	EPATC	0.0036	0.0052	0.0069
	PAIO	0.0036	0.0052	0.0069
μ_n	EPATC	0.8619	0.8553	0.8459
	PAIO	0.8619	0.8553	0.8459
σ_n	EPATC	0.0170	0.0230	0.0286
	PAIO	0.0170	0.0230	0.0286

5. Conclusions

In this paper, an enhanced probabilistic analytic cascading approach is proposed to design multilevel multidisciplinary systems especially with interrelated responses between different levels. Such a correlation due to a common uncertainty source has an impact on both the statistical measures of upper-level performances and the final optimal solutions. In the EPATC approach, a more general PATC formulation that considers the covariance between the correlated responses in addition to their first two statistical moments is developed. A modified updating strategy is proposed to estimate the upper-level performance considering the covariance between the correlated responses. The proposed EPATC approach is tested via a mathematical example and a multiscale design problem to demonstrate its applicability and effectiveness. Based on our empirical study, the EPATC approach outperforms the PATC method especially when the correlation between interrelated responses has a large impact on the statistical measures of system performances and the optimal solution of the hierarchical system designs.

Acknowledgments

The grant support from National Science Foundation (CMMI – 0522662) and the China Scholarship Council are greatly acknowledged. The views expressed are those of the authors and do not necessarily reflect the views of the sponsors.

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