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Including the Effects of Future Tests in Aircraft Structural Design

Erdem Acar[1], Raphael T. Haftka[2], Nam-Ho Kim[2], Deepti Buchi[2]

[1] TOBB University of Economics and Technology, Söğütözü, Ankara, Turkey, acar@etu.edu.tr
[2] University of Florida, Gainesville, Florida, USA, haftka@ufl.edu, nkim@ufl.edu, buchi@ufl.edu

1. Abstract

In this paper, we investigate the effects of future tests on aircraft structural safety, focusing on the numbers of coupon tests and structural element tests. The mean failure stress is assumed to be predicted by a failure criterion (e.g. Tsai-Wu), and the initial distribution of this mean failure stress reflects the uncertainty in the analysis procedure that uses coupon test data to predict structural failure. In addition to the uncertainty in the mean failure stress, there is also uncertainty in its variability due to the finite number of coupon tests. The Bayesian technique is used to update the failure stress distribution based on results of the element tests. We consider structural design based on point stress analysis following the FAA regulations (using B-basis allowables), and show tradeoffs between the number of tests and the weight of the structure for a given probability of failure. We found that element tests are more influential than coupon tests. This indicates that aircraft companies may reduce the number of coupon tests by moving to reliability-based design optimization (RBDO).

2. Keywords: future tests; aircraft safety; structural design; uncertainty reduction;

3. Introduction

In reliability based design optimization, it is customary to design for given uncertainty. However, this is unrealistic, because after the design of a structural component it is customary to engage in vigorous uncertainty reduction activities such as structural tests. It would be therefore beneficial to include the effects of planned tests in the design process. It may be even advantageous to design tests together with the structure, trading off the cost of more weight against additional tests. It is challenging, though to model the effect of future tests. The objective of the present paper is to explore efficient modeling of the effect of future tests on uncertainty in structural failure predictions. Four quartiles of the failure stress distributions are modeled as normal distributions. Then these quartiles are used to fit a Johnson distribution to the failure stress. That is, the failure stress is represented as a Johnson distribution, whose parameters are themselves distributions that depend on the quality of the tests.

We investigate in particular the effect of the number of coupon tests and number of structural element tests on the final distribution of the failure stress. The initial distribution is assumed to be obtained by a failure criterion such as Tsai-Wu and results of coupon tests. Bayesian techniques update the failure stress distribution from the results of the element tests.

Finally, we consider structural design following the FAA regulations using B-basis allowables. We show tradeoffs between the number of tests and the weight of the structure for a given probability of failure. These could allow a designer to choose between additional tests and heavier weight depending on the cost of testing and the cost of carrying the additional weight.

Section 4 discusses the safety measures taken during aircraft structural design. Section 5 presents a simple uncertainty classification that distinguishes uncertainties that affect an entire fleet (errors) from uncertainties that vary from one aircraft to another in the same fleet (variability). Section 6 discusses modeling of errors and variability throughout the design and testing of an aircraft. Section 7 describes probability of failure estimation via Monte Carlo simulations. Finally, the results and the concluding remarks are given in Sections 8 and 9, respectively.

4. Safety measures

The safety of aircraft structures is achieved by designing these structures to operate well in the presence of uncertainties and taking steps to reduce the uncertainties. The following gives brief description of these safety measures.

4.1. Safety measures for designing structures under uncertainties

Load Safety Factor: In transport aircraft design, FAA regulations state the use of a load safety factor of 1.5 (FAR-25.303 [2]). That is, aircraft structures are designed to withstand 1.5 times the limit load without failure.

Conservative Material Properties: To account for uncertainty in material properties, FAA regulations state the use of conservative material properties (FAR-25.613 [3]). These are characterized as A-basis and/or B-basis material property values. Detailed information on these values is provided in Volume 1, Chapter 8 of the Composite Materials Handbook [4]. We use B-basis values, which are.determined by calculating the value of a material property exceeded by 90% of the population with 95% confidence. The basis values are determined by testing a number of coupons selected randomly from a material batch. In this paper, B the nominal value of the number of coupon tests is taken 50. The effect of the number of coupon tests is also explored.

Other measures such as redundancy are not modeled here.

4.2. Safety measures for reducing uncertainties

Improvements in accuracy of structural analysis and failure prediction of aircraft structures reduce errors and enhance the level of safety of the structures. These improvements may be due to better modeling techniques, more detailed finite element models made possible by faster computers, or more accurate failure theories. Similarly, the variability in material properties can be reduced through quality control and improved manufacturing processes. Variability reduction in damage and ageing effects is accomplished through inspections and structural health monitoring. Refer to the papers by Acar et al. [5] for effects of error reduction, Qu et al. [6] for effects of variability reduction, and Acar et al. [7] for effects of reduction of both error and variability.

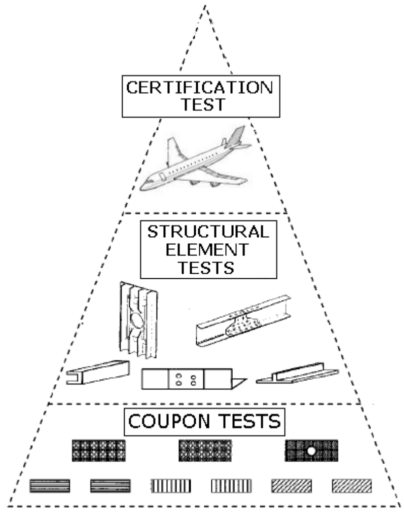

Figure 1: Simplified three-level tests

In this paper, we focus on error reduction through structural tests, while other uncertainty reduction measures are left out for future studies. Structural tests are conducted in a building block procedure (Volume I, Chapter 2 of [4]). First, individual coupons are tested to estimate the mean and variability in failure stress. The mean structural failure is estimated based on failure criteria (such as Tsai-Wu) and this estimate is further improved using element tests. Then a sub-assembly is tested, followed by a full-scale test of the entire structure. Here we use the simplified three-level test procedure depicted in Figure 1, which includes coupon tests, structural element tests and final certification test.

The first level is the coupon tests, where coupons (i.e., material samples) are tested to estimate failure stress. The FAA regulation FAR 25-613 requires aircraft companies to perform “enough” tests to establish design values of material strength properties (B-basis value). As the number of coupon tests increases, the errors in the assessment of the material properties are reduced. However, since testing is costly, the number of coupon tests is limited to about 30-80 for B-basis value calculation.

At the second level, structural elements are tested. The main target of element tests is to reduce errors related to failure theories (e.g., Tsai-Wu) used in predicting failure of structural elements. Here, the nominal value of the number of structural element tests is taken as 3.

At the uppermost level, certification (or proof) testing of the overall structure is conducted (FAR 25-307 [8]). It is intended to reduce the chance of failure in flight due to errors in the structural analysis of the overall structure (e.g., finite element errors, errors in failure mode prediction). While failure in flight often has fatal consequences, certification failure often has serious financial implications. So we measure the success of the tests in terms of their effects on both probability of failure in flight probability of certification failure.

5. Structural uncertainties

A good analysis of different sources of uncertainty in engineering simulations is provided by Oberkampf *et al.* [9, 10]. To simplify, we distinguish only between errors (uncertainties that apply equally to the entire fleet of an aircraft model) and variability (uncertainties that vary for the individual aircraft) as we used in [11-12]. The distinction, presented in Table 1, is important because safety measures usually target either errors or variability. While variabilities are random uncertainties that can be readily modeled probabilistically, errors are fixed for a given aircraft model (e.g., Boeing 737-400) but they are largely unknown.. We model errors probabilistically by using uniform distributions to maximize the entropy. To simulate errors, we assume that we have a large number of nominally identical aircraft being designed (e.g., by Airbus, Boeing, Embraer, Bombardier, etc.), with the errors being fixed for each aircraft.

6. Modeling errors and variability

6.1. Errors in estimating material strength properties from coupon testing

Coupon tests are conducted to obtain the statistical characterization of material strength properties, such as

Table 1: Uncertainty Classification

Type of Uncertainty	Spread	Cause	Remedies
Error (mostly epistemic)	Departure of the average fleet of an aircraft model (e.g. Boeing 737-400) from an ideal	Errors in predicting structural failure, construction errors, deliberate changes	Testing and simulation to improve the mathematical model and the solution
Variability (aleatory)	Departure of an individual aircraft from fleet level average	Variability in tooling, manufacturing process, and flying environment	Improvement of tooling and construction. Quality control

failure stress, and their corresponding design values (A-basis or B-basis). With a finite number n_c of coupon tests, the statistical characterization involves errors. Therefore, the calculated values of the mean and the standard deviation of the failure stress will be uncertain. We assume that the failure stress follows normal distribution, so the calculated mean also follows normal distribution. In addition, when n_c is larger than 25, the distribution of the calculated standard deviation tends to be normal. Then, the calculated failure stress can be expressed as

$$\left(\sigma_{cf}\right)_{calc} = Normal\left[\left(\bar{\sigma}_{cf}\right)_{calc}; Std\left(\sigma_{cf}\right)_{calc}\right] \tag{1}$$

where calculated mean and the calculated apparent standard deviation can be expressed as

$$\left(\bar{\sigma}_{cf}\right)_{calc} = Normal\left(\bar{\sigma}_f; \frac{Std\left(\sigma_f\right)}{\sqrt{n_c}}\right) \tag{2}$$

$$Std\left(\sigma_{cf}\right)_{calc} = Normal\left(Std\left(\sigma_f\right)\sqrt{\frac{1+\sqrt{\frac{n_c-3}{n_c-1}}}{2}}; Std\left(\sigma_f\right)\sqrt{\frac{1-\sqrt{\frac{n_c-3}{n_c-1}}}{2}}\right) \tag{3}$$

where $\bar{\sigma}_f$ and $Std(\sigma_f)$ are, respectively, the true values of the mean and standard deviation of failure stress. Note that Eqs. (1)–(3) describe a random variable coming from a distribution (normal) whose parameters are also random. In this paper, this will be referred to as a distribution of distributions.

The allowable stress at the coupon level, σ_{ca}, is computed from the failure stress calculated at the coupon level, $\left(\bar{\sigma}_{cf}\right)_{calc}$, by using a knockdown factor, k_d, as

$$\sigma_{ca} = k_d\left(\bar{\sigma}_{cf}\right)_{calc} \tag{4}$$

The knockdown factor k_d is specified by the FAA regulations (FAR). It is computed to achieve that 90% of the failure stresses (measured in coupon tests) must exceed the stress-basis value with 95% confidence. For normal distribution, the knockdown factor depends on the number coupon tests and the c.o.v. $\left(c_{cf}\right)_{calc}$ of the failure stress as

$$k_d = 1 - k_B\left(c_{cf}\right)_{calc} \tag{5}$$

. The tolerance coefficient k_B is a function of the number of coupon tests n_c as given in [4] (Volume 1, Chapter 8, page 84) as

$$k_B \approx 1.282 + \exp\left(0.958 - 0.520\ln(n_c) + \frac{3.19}{n_c}\right) \tag{6}$$

6.2. Errors in structural element strength predictions

In the second level of testing structural elements are tested to validate the accuracy of the failure criterion used (e.g., Tsai-Wu). Here, we assume that structural element tests are conducted for a specified combination of loads corresponding to critical loading. For this load combination, the failure surface can be boiled down to a single failure stress $\bar{\sigma}_{ef}$ where the subscript 'e' stands for structural element tests.

If the failure criterion used to predict the failure were perfect, and we performed infinite number of coupon tests, then we could predict the true mean element failure stress at the structural element tests. The actual value would vary only due to material variability. However, neither condition is satisfied, so we introduce an error e_{ef} in the calculated value of the mean failure stress at the element level is

$$\left(\bar{\sigma}_{ef}\right)_{calc}=\left(1-e_{ef}\right)\left(\bar{\sigma}_{cf}\right)_{calc} \quad (7)$$

The sign in front of the error term is negative, since we formulate the error expressions such that a positive error implies a conservative decision. The initial distribution of $\left(\bar{\sigma}_{ef}\right)_{calc}$ is obtained by estimate of the error e_{ef} and using the results of coupon tests $\left(\bar{\sigma}_{cf}\right)_{calc}$. The information from element tests is used by a Bayesian procedure to update the failure stress distribution (see [13] for details). In practice, simpler procedures are often used, such as selecting the lowest failure stress from element tests. Therefore, our assumption will tend to overestimate the beneficial effect of element tests.

The allowable stress based on the element test is calculated from

$$\sigma_{ea}=k_d\left(\bar{\sigma}_{ef}\right)_{calc}^{updated} \quad (8)$$

where the $\left(\bar{\sigma}_{ef}\right)_{calc}^{updated}$ is the value of the mean failure stress corresponding to the maximum PDF.

Redesign based on element tests:

Besides updating the failure stress, element tests lead to design changes if the design is unsafe or overly conservative. That is, if large or small failure stress values are obtained from the element tests, the company may increase or reduce the thicknesses of the elements. We did not find published data on redesign practices, and so we devised a common sense approach reflected in Table 2. We assumed that if the B-basis value obtained after element tests, σ_{ea}, is more than 5% higher than the B-basis value obtained from coupon tests, σ_{ca}, then the element thickness is reduced by σ_{ca}/σ_{ea} ratio. If the B-basis value obtained after element tests is more than 2% lower than the B-basis value obtained from coupon tests, the element thickness is increased by σ_{ca}/σ_{ea} amount. This lower tolerance reflects the need for safety. Otherwise, no redesign is performed. The thickness of the element is obtained through

$$t_{elem}=(1-e_{ef})\frac{S_F P_d}{w}\frac{1}{\sigma_{ea}} \quad \text{(no redesign)}$$

$$t_{elem}=(1-e_{ef})\frac{S_F P_d}{w\sigma_{ea2}}\frac{1.01}{C.F.}=(1-e_{ef})\frac{S_F P_d}{w}\frac{1.01\sigma_{ea}}{\sigma_{ca}\sigma_{ea2}} \quad \text{(redesign)} \quad (8)$$

where S_F is the load safety factor, P_d is the design load for testing the elements. Since redesign requires new elements to be built and tested, it is costly. Therefore, we do not repeat the tests. To protect against uncertainties in the test of the redesigned element we have an additional 1% reduction in the calculated allowable value (provided by the term 1.01 in Eq. (8)).

Table 2: Simulation of element tests

1. Generate random numbers for the quartiles of the mean failure stress
2. Calculate the B-basis value using the quartiles
3. Check to see if redesign is needed
4. If redesign is needed
    a. Generate new random numbers for the quartiles
    b. Calculate the new B-basis value using the new quartiles
5. Compute the design thickness of the element using the B-basis value calculated
6. Compute probabilities of failure in certification tests and under service loads

6.3. Errors in structural strength predictions

Due to the complexity of the overall structural system, there will be errors in failure prediction of the overall structure that we denote as e_f. Therefore, the calculated value of the failure stress of the overall structure, $\left(\bar{\sigma}_f\right)_{calc}$, can be expressed as

$$\left(\bar{\sigma}_f\right)_{calc}=\left(1-e_f\right)\left(\bar{\sigma}_{ef}\right)_{calc} \quad (9)$$

The allowable stress at the structural design level, σ_a, can be related to the allowable stress computed at the element level, σ_{ea}, through the following relation

$$\sigma_a=k_f\left(1-e_f\right)\sigma_{ea} \quad (10)$$

where k_f is an additional knockdown factor used at the structural level as an extra precaution. The value of k_f was taken 0.95 in this paper.

6.4. Errors in design

We consider static point stress design for simplicity. Other types of failures such as fatigue, corrosion or crack instability are not taken into account. Before starting the structural design, aerodynamic analysis needs to be performed to determine the loads acting on the aircraft. However, the calculated design load value, P_{calc}, differs from the actual loading P_d under conditions corresponding to FAA design specifications (e.g., gust-strength specifications). Since each company has different design practices, the error in load calculation, e_p, is different from one company to another. The calculated design load P_{calc} is expressed in terms of the true design load P_d as

$$P_{calc} = (1+e_P)\,P_d \tag{11}$$

Besides the error in load calculation, an aircraft company may also make errors in stress calculation. We consider a small region in a structural part, characterized by a thickness t and width w, that resists the load in that region. The value of the stress in a structural part calculated by the stress analysis team, σ_{calc}, can be expressed in terms of the load values calculated by the load team P_{calc}, the design width w_{design}, and the thickness t of the structural part by introducing the term e_σ representing error in the stress analysis

$$\sigma_{calc} = (1+e_\sigma)\frac{P_{calc}}{w_{design}\,t} \tag{12}$$

Here we assume that the aircraft companies can predict stresses very accurately so that the effect of e_σ is negligible and is taken as zero. The calculated stress value is then used by a structural designer to calculate the design thickness t_{design} required to carry the calculated design load times the safety factor S_F. That is,

$$t_{design} = \frac{S_F P_{calc}}{w_{design}\sigma_a} = \frac{(1+e_P)}{(1-e_f)}\frac{S_F P_d}{w_{design}\,k_f\sigma_{ea}} \tag{13}$$

From Eq. (13), we can express the design value of the load carrying area as

$$A_{design} = t_{design} w_{design} = \frac{(1+e_P)}{(1-e_f)}\frac{S_F P_d}{k_f\sigma_{ea}} \tag{14}$$

6.5. Errors in construction

In addition to the above errors, there will also be construction errors in the geometric parameters. These construction errors represent the difference between the values of these parameters in an average airplane (fleet-average) built by an aircraft company and the design values of these parameters. The error in width, e_w, represents the deviation of the design width of the structural part, w_{design}, from the average value of the width of the structural part built by the company, $w_{built-av}$. Thus,

$$w_{built-av} = (1+e_w)\,w_{design} \tag{15}$$

Similarly, the built thickness value will differ from its design value such that

$$t_{built-av} = (1+e_t)\,t_{design} \tag{16}$$

Then, the built load carrying area $A_{built-av}$ can be expressed using the first equality of Eq. (14) as

$$A_{built-av} = (1+e_t)(1+e_w)\,A_{design} \tag{17}$$

Table 3 presents nominal values for the errors assumed here.

Table 3: Distribution of error terms and their bounds

Error factors	Distribution Type	Mean	Bounds
Error in load calculation, e_P	Uniform	0.0	± 10%
Error in width, e_w	Uniform	0.0	± 1%
Error in thickness, e_t	Uniform	0.0	± 3%
Error in failure prediction, e_f	Uniform	0.0	± 10%
Error in failure prediction, e_{ef}	Uniform	0.0	± 10%

The errors here are modeled by uniform distributions, following the principle of maximum entropy. For instance, the error in the built thickness of a structural part (e_t) is defined in terms of the error bound $(b_t)_{built}$ using

$$e_t = U\left[0,(b_t)_{built-av}\right] \tag{18}$$

Here 'U' indicates that the distribution is uniform, '0 (zero)' is the average value of e_t, and the error bound is

$(b_t)_{built-av}$ =0.03. Hence, the lower bound for the thickness value is the average value minus 3% of the average and the upper bound for the thickness value is the average value plus 3% of the average.

6.6. Total error

The expression for the built load carrying area, $A_{built-av}$, of a structural part can be reformulated by combining Eqs. (14) and (17) as

$$A_{built-av} = (1+e_{total})\frac{S_F P_d}{k_f \sigma_{ea}} \quad (19)$$

where

$$e_{total} = \frac{(1+e_P)(1+e_t)(1+e_w)}{(1-e_f)} - 1 \quad (20)$$

Here e_{total} represents the cumulative effect of the individual errors on the load carrying capacity of the structure.

6.7. Variability

In the previous sections, we analyzed the different types of errors made in the design and construction stages, representing the differences between the fleet average values of geometry, material and loading parameters and their corresponding design values. For a given design, these parameters vary from one aircraft to another in the fleet due to variability in tooling, construction, flying environment, etc. For instance, the actual value of the thickness of a structural part, $t_{built-\text{var}}$, is defined in terms of its fleet average built value, $t_{built-av}$, by

$$t_{built-\text{var}} = (1+v_t)\,t_{built-av} \quad (21)$$

We assume that v_t has a uniform distribution with 3% bounds (see Table 4). Then, the actual load carrying area $A_{built-\text{var}}$ can be defined as

$$A_{built-\text{var}} = t_{built-\text{var}} w_{built-\text{var}} = (1+v_t)(1+v_w)\,A_{built-av} \quad (22)$$

where v_w represents effect of the variability on the fleet average built width.

Table 4 presents the assumed distributions for variabilities. Thickness error in Table 3 is uniformly distributed with bounds of ±3%. Thus the difference between all thicknesses over the fleets of all companies is up to ±6%. However, the combined effect of the uniformly distributed error and variability is not uniformly distributed.

Table 4: Distribution of random variables having variability

Variables	Distribution type	Mean	Scatter
Actual service load, P_{act}	Lognormal	P_d = 2/3	10% c.o.v.
Actual built width, $w_{built-\text{var}}$	Uniform	$w_{built-av}$	1% bounds
Actual built thickness, $t_{built-\text{var}}$	Uniform	$t_{built-av}$	3% bounds
Failure stress, σ_f	Normal	1.0	8% c.o.v.
v_w	Uniform	0	1% bounds
v_t	Uniform	0	3% bounds

c.o.v.= coefficient of variation

6.8. Certification test

After a structural part has been built with random errors in stress, load, width, allowable stress and thickness, it may fail in certification testing part of the airplane. Recall that the structural part will not be manufactured with complete fidelity to the design due to variability in the geometric properties. That is, the actual values of these parameters $w_{built-\text{var}}$ and $t_{built-\text{var}}$ will be different from their fleet-average values $w_{built-av}$ and $t_{built-av}$ due to variability. The structural part is then loaded with the design axial force of S_F times P_{calc}, and if the stress exceeds the failure stress of the structure σ_f, then the structure fails and the design is rejected; otherwise it is certified for use. That is, the structural part is certified if the following inequality is satisfied

$$\sigma - \sigma_f = \frac{S_F P_{calc}}{w_{built-\text{var}} t_{built-\text{var}}} - \sigma_f \le 0 \quad (23)$$

7. Probability of Failure Calculation by Separable MCS

To calculate the probability of failure, we incorporate the statistical distributions of errors and variability in a Monte Carlo simulation. Errors are uncertain at the time of design, but do not change for individual realizations (in actual service) of a particular design. On the other hand, all individual realizations of a particular design are different due to variability. The simulation of error and variability can be easily implemented through a two-level

Monte Carlo simulation [11]. At the upper level different aircraft companies can be simulated by assigning random errors to each, and at the lower level we simulated variability in dimensions, material properties, and loads related to manufacturing variability and variability in service conditions

The effect of element tests on failure stress distribution is modeled using Bayesian updating. If the Bayesian updating is used directly within an MCS loop for design thickness determination, the computational cost will be very high. In this paper, instead, the Bayesian updating is performed aside in a separate MCS, before starting with the MCS loop for design thickness determination. The procedure followed for Bayesian updating can be described briefly as follows. First, the four quartiles of the mean failure stress are modeled as normal distributions. Then, these quartiles are used to fit a Johnson distribution to the mean failure stress. That is, the mean failure stress is represented as a Johnson distribution, whose parameters are themselves distributions that depend on the quality of the tests. Finally, Bayesian updating is used to update the mean failure stress distribution. Details of this procedure can be found in the appendix of [14].

The prediction of probability of failure via the conventional Monte Carlo procedure requires trillions of simulations for the level of 10^{-7} failure probability. In order to address the computational burden, separable Monte Carlo procedure is used [12, 15]. This procedure applies when the failure condition can be expressed as $g_1(x_1)>g_2(x_2)$, where x_1 and x_2 are two disjoint sets of random variables. To take advantage of this procedure, we formulate the failure condition in a separable form, so that g_1 will depend only on variabilities and g_2 only on errors. The common formulation of the structural failure condition is in the form of a stress exceeding the material limit. This form, however, does not satisfy separability. For example, the stress depends on variability in material properties as well as design area, which reflects errors in the analysis process. To bring the failure condition to the right form, we instead formulate it as the required cross sectional area A'_{req} being larger than the built area $A_{built-av}$, as given in Eq (24).

$$A_{built-av} < \frac{A_{req}}{(1+v_t)(1+v_w)} \equiv A'_{req} \tag{24}$$

where A_{req} is the cross-sectional area required to carry the actual loading conditions for a particular copy of an aircraft model, and A'_{req} is what the built area (fleet-average) needs to be in order for the particular copy to have the required area after allowing for variability in width and thickness.

$$A_{req} = P/\sigma_f \tag{25}$$

The required area depends only on variability, while the built area depends only on errors. When certification testing is taken into account, the built area, $A_{built-av}$, is replaced by the certified area, A_{cert}, which is the same as the built area for companies that pass certification. However, companies that fail are not included. That is, the failure condition is written as

$$\text{failure without certification tests:} \quad A_{built-av} - A'_{req} < 0 \tag{26a}$$

$$\text{failure with certification tests:} \quad A_{cert} - A'_{req} < 0 \tag{26b}$$

8. Results

The effects of the number of coupon tests, the number of element tests, redesign of element tests and certification test are reported. As noted earlier, the nominal values of the number of coupon tests and the number of element tests are 50 and 3, respectively. The redesign of element tests and the certification test are included in the analysis except for the cases that investigate the effect of redesign of element tests and the certification test.

8.1. Effect of the number of coupon tests

The effects of increasing the number of coupon tests on the thickness and probability of failure are presented in Table 6. The thickness values provided in Table 6 are based on the load and material property values assumed in Table 5. As the number of coupon tests increases, the mean thickness is reduced (since B-basis value is increased), and the coefficient of variation of the thickness is reduced (since the coefficient of variation of the B-basis value is reduced). These two reductions have opposing effect on the probability of failure, and the probability of failing certification. However, the net effect is that both probabilities increase, indicating. that the knockdown factor used by the FAA to compensate for small number of coupon tests (Eq. 6) is conservative, so performing more tests, actually makes the aircraft less safe!

To provide an indication of the accuracy of the numbers in Table 6, simulations are rerun with a different seed for the random number generator. Regenerated results are provided in Table 7. We see that the mean stress results are accurate to the fourth digit, while the probabilities are only accurate to the second digit.

Table 6: Effects of the number of coupon tests. *Number of element tests, n_c, is 3. Redesign of element tests and certification test are included in the analysis.*

n_c	t_{mean}	t_{cov}	P_f	PFCT*
30	1.268	0.116	1.02×10^{-4}	0.0576
50	1.253	0.114	1.27×10^{-4}	0.0654
80	1.245	0.113	1.44×10^{-4}	0.0711

* PFCT: Probability of failing in certification tests

Table 7: Regeneration of Table 6 results by using a different seed for the random number generator.

n_c	t_{mean}	t_{cov}	P_f	PFCT*
30	1.268	0.116	1.03×10^{-4}	0.0566
50	1.254	0.114	1.23×10^{-4}	0.0664
80	1.246	0.113	1.38×10^{-4}	0.0702

* PFCT: Probability of failing in certification tests

Since the mean thickness reduces as the number of coupon tests increases, the aircraft builder may decide to keep the mean thickness constant. This can be achieved by adjusting the knockdown factor k_f in Eq. (13) so as to have same mean thickness for different number of coupon tests. First, the knockdown factor k_f is varied by -10%, -5%, 5%, and 10% of its nominal value and simulations are performed. Then, response surfaces are constructed for the mean thickness (t_{mean}), the probability of failure (P_f), and the probability of failing in certification test (*PFCT*) for each value of number of coupon tests. Finally, P_f and *PFCT* values corresponding to the mean thickness value of 1.253 are computed. This practice also reduces the numerical noise in simulation results. Table 8 shows that increasing the number of coupon tests from 50 to 80 leads to 13% reduction in probability of failure, whereas reducing the number of coupon tests to 30 increases by 20% the probability of failure. We can also conclude that increasing the number of coupon tests reduces the probability of failure for the same weight, but the rate of reduction diminishes with the number of tests. Overall, it appears that increasing the number of coupon tests has only small effect on the probability of failure in service or on the probability of failing certification.

Table 8: Effects of the number of coupon tests for the same probability of failure. *Number of element tests, n_e, is 3. Redesign of element tests and certification test are included in the analysis.*

n_c	t_{mean}	P_f	PFCT*
30	1.256	1.27×10^{-4}	0.0666
50	1.253	1.27×10^{-4}	0.0654
80	1.250	1.27×10^{-4}	0.0670

* PFCT: Probability of failing in certification tests

8.2. Effect of the number of element tests

The effects of increasing the number of element tests on the thickness and probability of failure are presented in Table 9. Increasing the number of element tests does not have a significant effect on the mean thickness, but we just see some fluctuations in the mean thickness values due to numerical noise. The coefficient of variation of the thickness, on the other hand, is reduced significantly because of the reduction of the error term e_{ef}.

Table 9: Effect of the number of element tests. *Number of coupon tests, n_c, is 50. Redesign of element tests and certification test are included in the analysis.*

n_e	t_{mean}	t_{cov}	P_f	PFCT
0	1.244	0.119	1.81×10^{-4}	0.0880
1	1.257	0.119	1.37×10^{-4}	0.0714
2	1.254	0.115	1.29×10^{-4}	0.0676
3	1.253	0.114	1.27×10^{-4}	0.0654
4	1.253	0.112	1.20×10^{-4}	0.0637
5	1.252	0.111	1.18×10^{-4}	0.0636

To provide an indication of the accuracy of the numbers in Table 9, simulations are repeated with a different seed for the random number generator, as we did earlier for Table 6. We found that the mean stress results are accurate to the fourth digit, while the probabilities are only accurate to the second digit.

To analyze the probability of failure and weight tradeoffs, the probability of failure is fixed to 1.27×10^{-4}, which corresponds to performing three element tests and fifty coupon tests (the nominal values). Table 10 shows that if we dispense with element tests, then we will need to put 1% extra weight to achieve to the same probability of failure.

Comparing the weight difference between 30 and 80 coupon tests (which is half a percent), to the weight difference between 0 and 3 element tests (which is 1 percent), we see that the element tests are more influential. This indicates that moving the RBDO may allow aircraft companies to reduce the number of coupon tests.

Table 10. Effects of the number of element tests for the same probability of failure. *Number of coupon tests, n_c, is 50. Redesign of element tests and certification test are included in the analysis.*

n_e	t_{mean}	*% increase in thickness*	P_f	*PFCT*
0	1.266	1.0	1.27×10^{-4}	0.0653
1	1.263	0.8	1.27×10^{-4}	0.0656
2	1.256	0.2	1.27×10^{-4}	0.0668
3	1.253	---	1.27×10^{-4}	0.0654
4	1.250	-0.2	1.27×10^{-4}	0.0672
5	1.249	-0.3	1.27×10^{-4}	0.0672

The probability of failing in the certification tests is likely a big motivator for the aircraft companies, hence we also investigate how much extra weight would be needed to maintain the probability of failing in certification test if the company intends to eliminate the element tests. Table 11 shows that if a company aims to eliminate the element tests, the structural weight must be increased by 1% to achieve to the same probability of failing in certification tests.

Table 11. Effects of the number of element tests for the same probability of failing in certification test. *Number of coupon tests, n_c, is 50. Redesign of element tests and certification test are included in the analysis.*

n_e	t_{mean}	*% increase in thickness*	P_f	*PFCT*
0	1.266	1.0	1.28×10^{-4}	0.0654
1	1.263	0.8	1.27×10^{-4}	0.0654
2	1.257	0.3	1.24×10^{-4}	0.0654
3	1.253	---	1.27×10^{-4}	0.0654
4	1.252	-0.1	1.22×10^{-4}	0.0654
5	1.251	-0.2	1.22×10^{-4}	0.0654

8.3. Effect of the certification test

Finally, the effect of certification test on the mean thickness and reliability are explored. Table 12 shows that if certification is not performed, then the mean thickness is reduced by a small amount while the coefficient of variation of the thickness is increased significantly. Therefore, the probability of failure is increased by almost twice. Even if the mean thickness is adjusted to its nominal value, the probability of failure is 54% larger! The overall conclusion is that the certification test is very effective of maintaining the reliability.

Table 12: Effects of certification test. *Number of coupon tests=50. Number of element tests=3.*

	t_{mean}	t_{cov}	P_f
Certification	1.253	0.114	1.27×10^{-4}
No certification	1.244	0.119	2.31×10^{-4}
No certification with adjusted mean thickness	1.253	0.119	1.94×10^{-4}

9. Concluding remarks

The effects of aircraft structural tests on aircraft structural safety were explored. In particular, we studied the effects of the number of coupon tests and the number of structural element tests on the weight and safety of the design. We simulated a structural design following the FAA regulations and drew from the results the following conclusions.

- As the number of coupon tests is increased, the mean allowable stress increases so the mean thickness reduces. While the standard deviation of the thickness decreases, the probability of failure increases as does the probability of failing certification. This indicates that the FAA knockdown factor for compensating for small number of coupon tests is conservative.

- As the number of element tests is increased, the probability of failure reduces for the same weight. If we dispense with element tests, then we will need to put about 1% extra structural weight to achieve to the same probability of failure.
- Element tests are found to be more influential than coupon tests, indicating that moving the RBDO may allow aircraft companies to reduce the number of coupon tests.
- If certification test is not performed, the probability of failure is increased by 54%, so the certification test is an effective way of maintaining the reliability.

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