Including the Effects of Future Tests in Aircraft Structural Design

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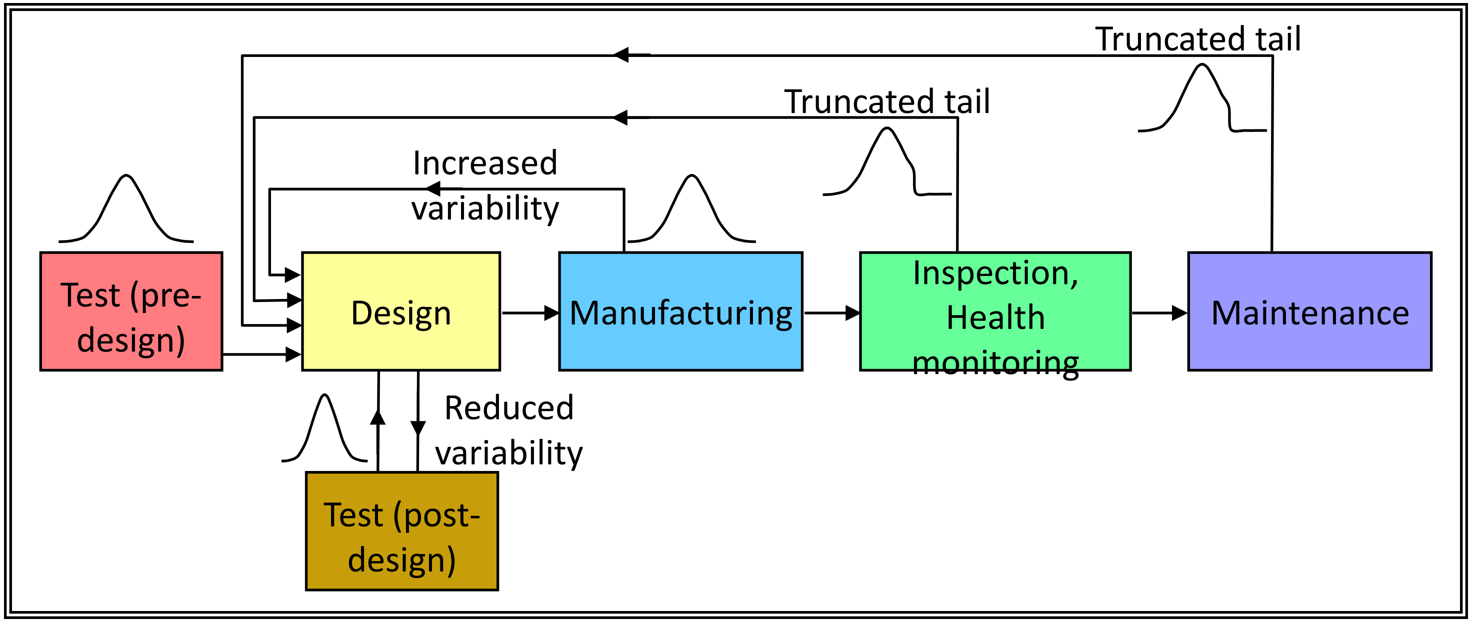
Motivation

- In reliability estimation of aircraft, all uncertainties available at the design stage are considered.
- However, the actual aircraft is much safer, because after the design many uncertainty reduction activities are performed.



- We analyze the tradeoffs between tests, reliability and weight.
  - A first step towards simultaneous design of structure and tests

Outline

- Safety measures
- Structural uncertainties
  - o errors
  - o variability
- Modeling uncertainties throughout design, construction and in-service use
  - Material strength predictions from coupon tests
  - Structural element strength predictions
  - Structural strength predictions
  - Errors in design and construction
  - Variabilities
- Reliability estimation using Monte Carlo simulations
- Results
- Concluding remarks

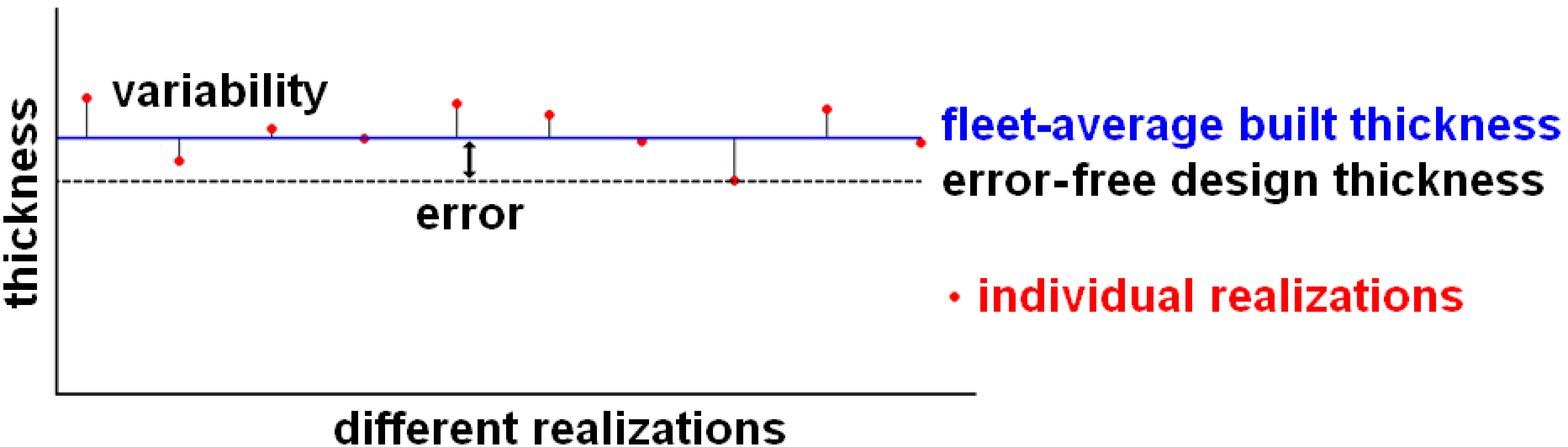
Safety measures

- Conservative design practices
  - Load safety factor of 1.5 (FAR-25.303)
  - Conservative material properties (FAR-25.613)
  - Redundancy

- Uncertainty reduction
  - o structural testing
  - o quality control
  - o inspection
  - o health monitoring
  - o maintenance
  - o post design improved analysis
  - o post design improved failure modeling

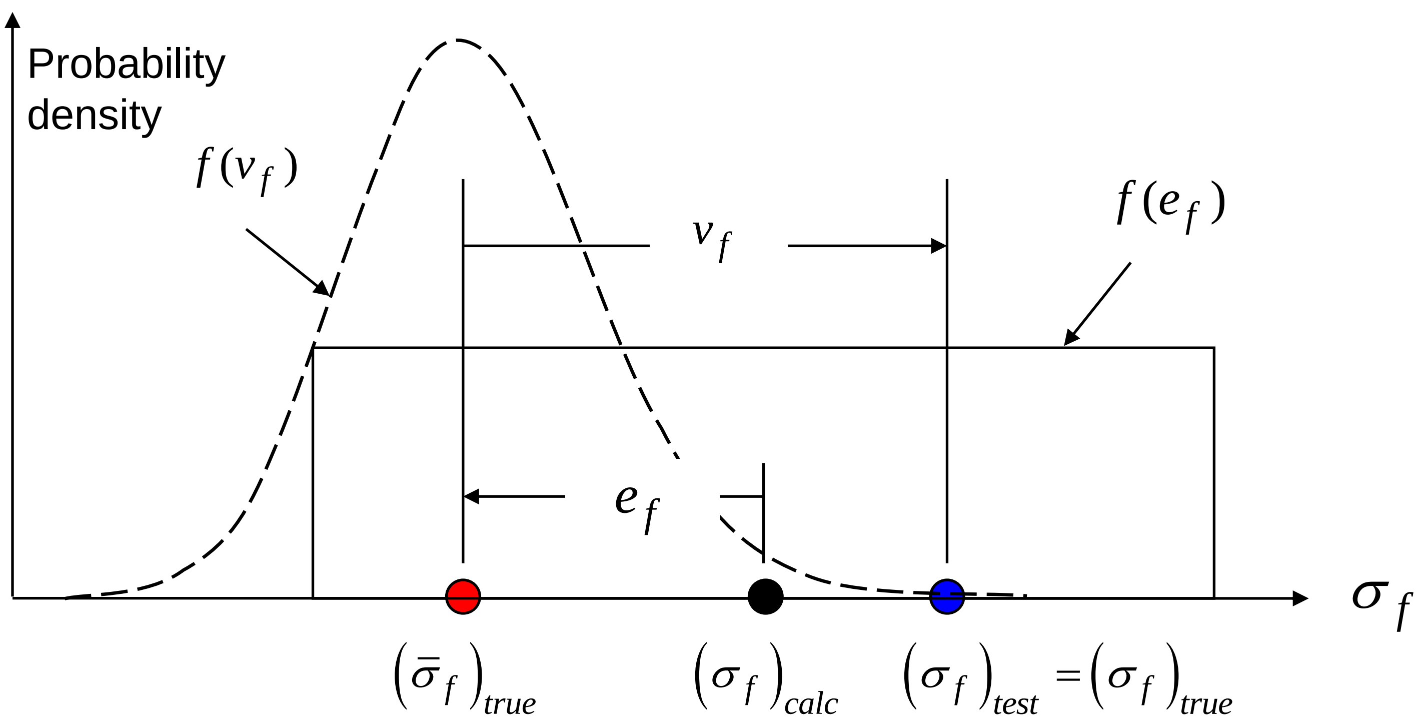
Uncertainty classification

Type	Spread	Cause	Remedies
Error (mostly epistemic)	Departure of fleet average from ideal	Errors in failure prediction Construction errors	Testing Improved accuracy
Variability (aleatory)	Departure of individual aircraft from fleet average	Manufacturing variability In service variability	Tighter tolerances Quality control



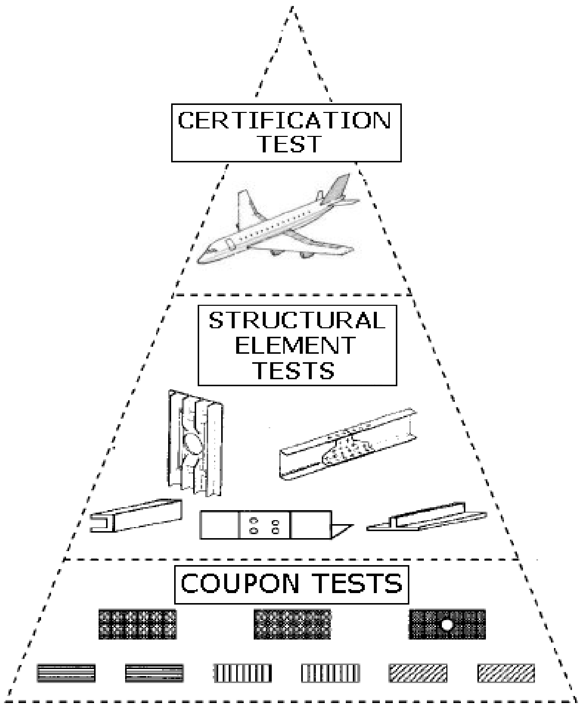
Error and variability in failure stress

$$\left(\sigma_f\right)_{test} = \left(1+v_f\right)\left(\bar{\sigma}_f\right)_{true} = \left(1+v_f\right)\left(1+e_f\right)\left(\sigma_f\right)_{calc}$$



Tests reduce uncertainty (due to errors)

- Aircraft structural tests are conducted in a building block procedure.
- Coupons are tested to estimate the mean and variability in failure stress.
- The mean structural failure is estimated based on failure criteria (such as Tsai-Wu) and this estimate is further improved using element tests.
- Components, sub-assemblies, assemblies are tested.
- Finally, full-scale test of the entire structure is conducted.



Simplified three-level tests

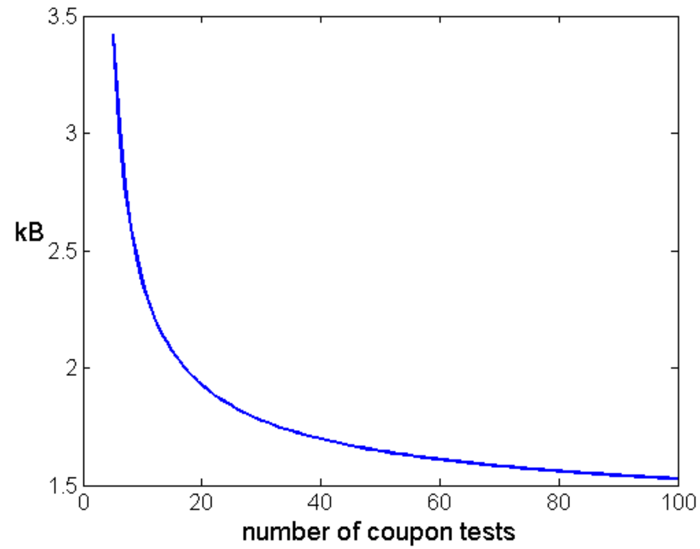
Errors in coupon testing

- Due to finite number of coupon tests, statistical characterization of strength has errors.
  - Mean and the standard deviation will be uncertain.

$$\left(\sigma_{cf}\right)_{calc} = Normal\left[\left(\bar{\sigma}_{cf}\right)_{calc}; Std\left(\sigma_{cf}\right)_{calc}\right]$$

$$\left(\bar{\sigma}_{cf}\right)_{calc} = Normal\left(\bar{\sigma}_f; \frac{Std\left(\sigma_f\right)}{\sqrt{n_c}}\right)$$

standard deviation distribution also close to Normal for N_{coupon}>25



- Allowable stress $\sigma_{ca} = k_d\left(\bar{\sigma}_{cf}\right)_{calc}$

  knock-down $k_d = 1 - k_B\left(c_{cf}\right)_{calc}$

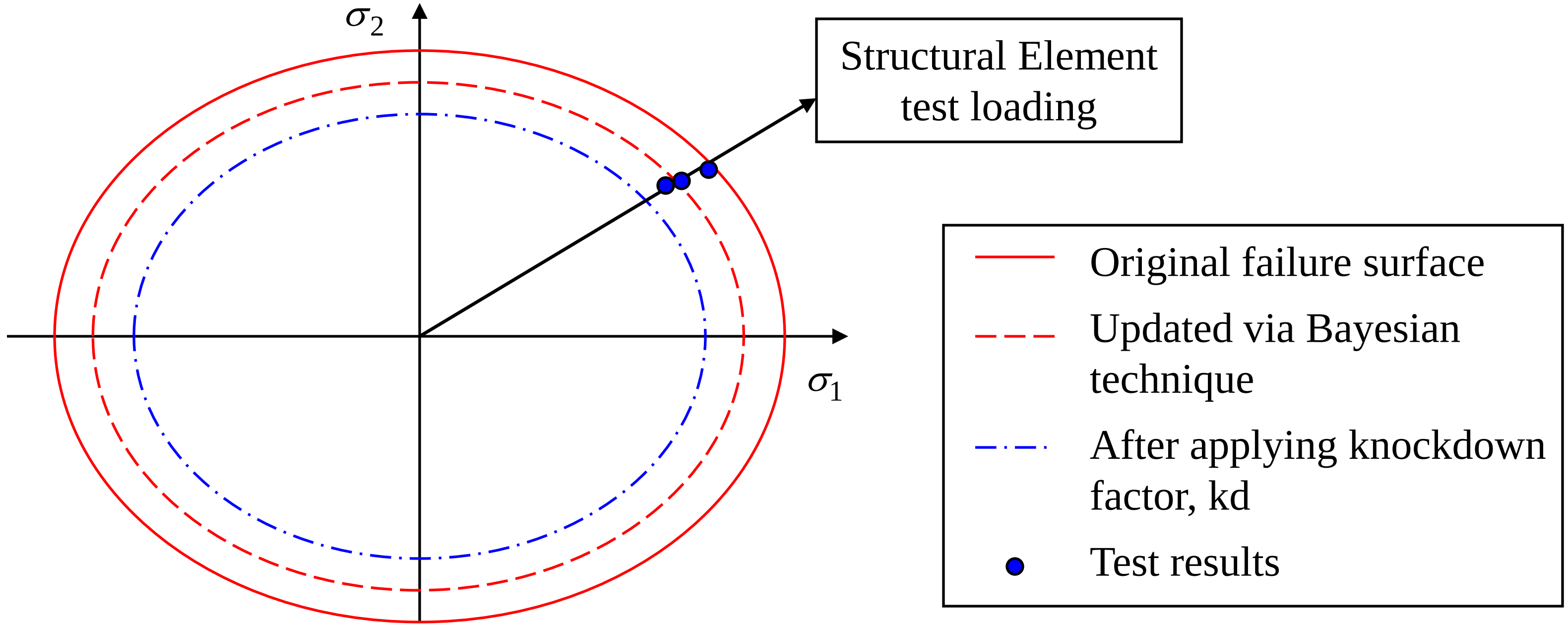
  Calculated c.o.v.

Using element tests to reduce error

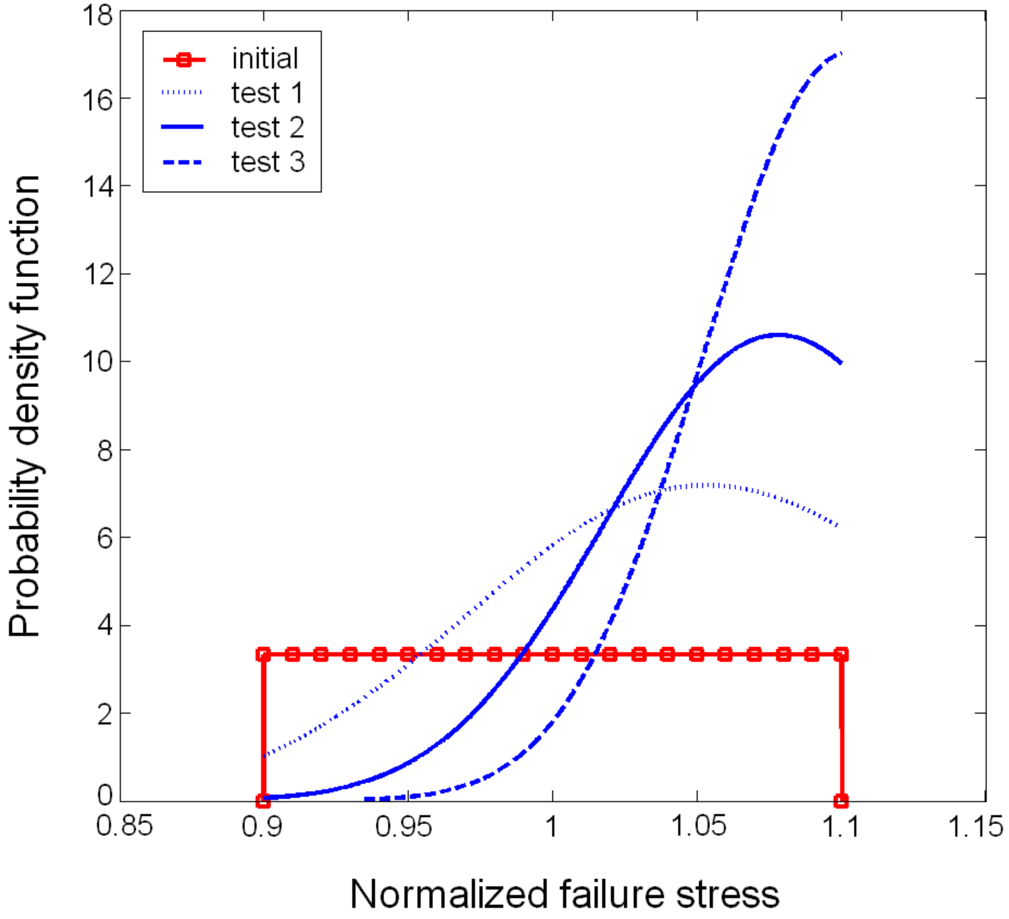
- **Calculated mean failure stress** $\left(\vec{\sigma}_{ef}\right)_{calc} = \left(1 - e_{ef}\right)\left(\bar{\sigma}_{cf}\right)_{calc}$

$$\sigma_{ea} = k_d \left(\bar{\sigma}_{ef}\right)_{calc}^{updated}$$

corresponds to the maximum PDF of the updated (Bayesian) distribution



Bayesian updating



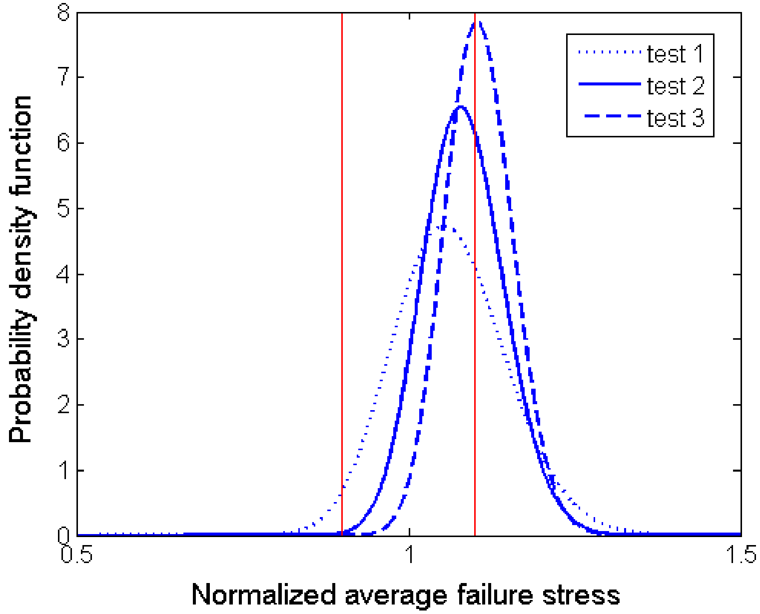
The initial distribution of mean failure stress: Uniform in [0.9, 1.1]

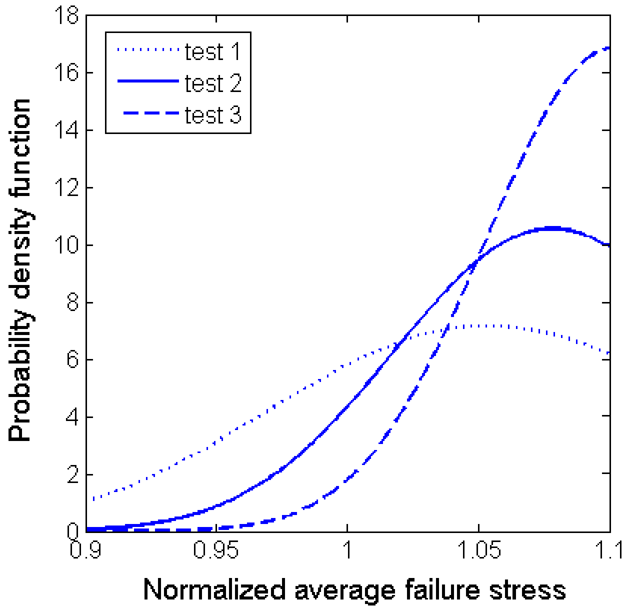
Three test results: 1.05, 1.10, 1.15.

$$f^{upd}\left(\bar{\sigma}_f\right)=\frac{f_{1,test}\left(\bar{\sigma}_f\right)f^{ini}\left(\bar{\sigma}_f\right)}{\int_{-\infty}^{\infty} f_{1,test}\left(\bar{\sigma}_f\right)f^{ini}\left(\bar{\sigma}_f\right)d\bar{\sigma}_f}$$

A Bayesian updating trick

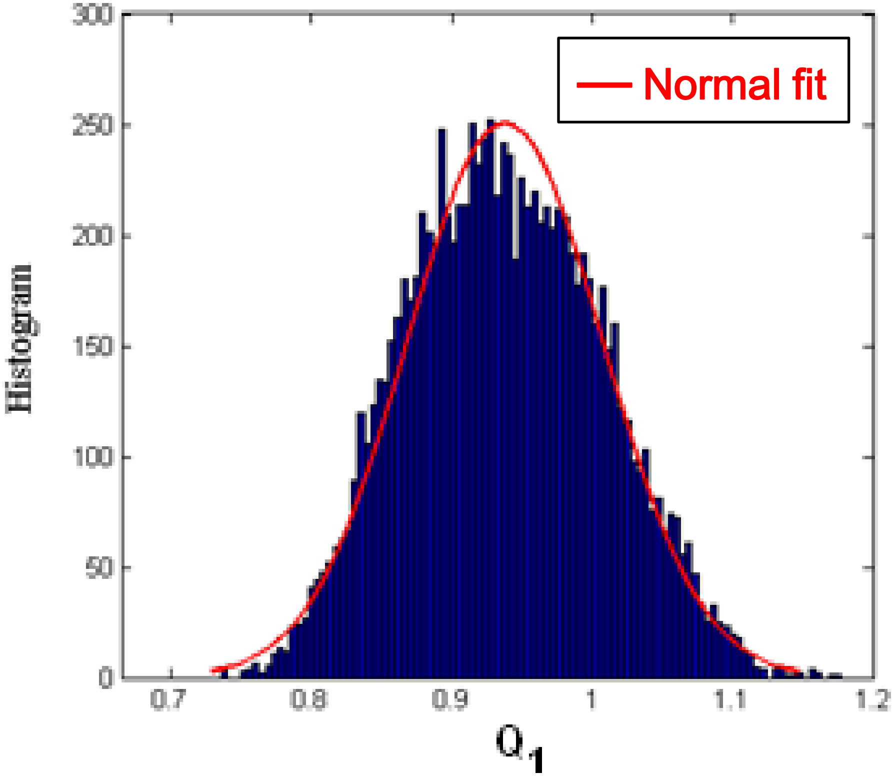
- Error bounds can be applied in Bayesian updating after updating for test results.

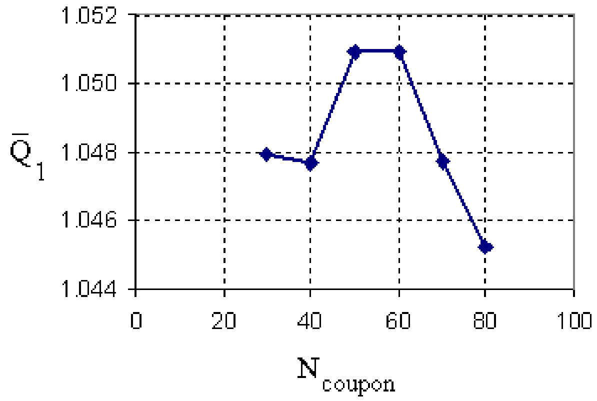


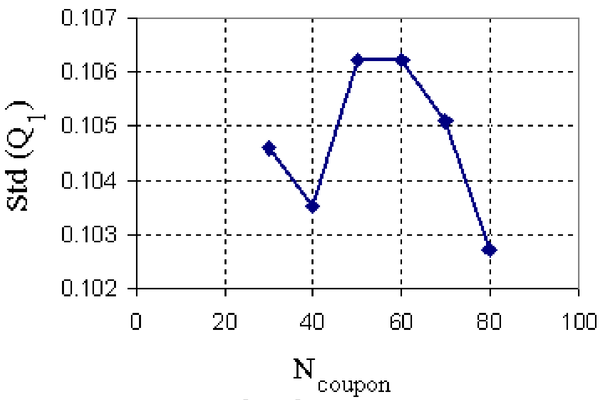


- Obtain good fit via "Johnson unbounded"
- So, the distribution for quartiles of Johnson is what we want.

Quartiles of mean failure stress after future element tests

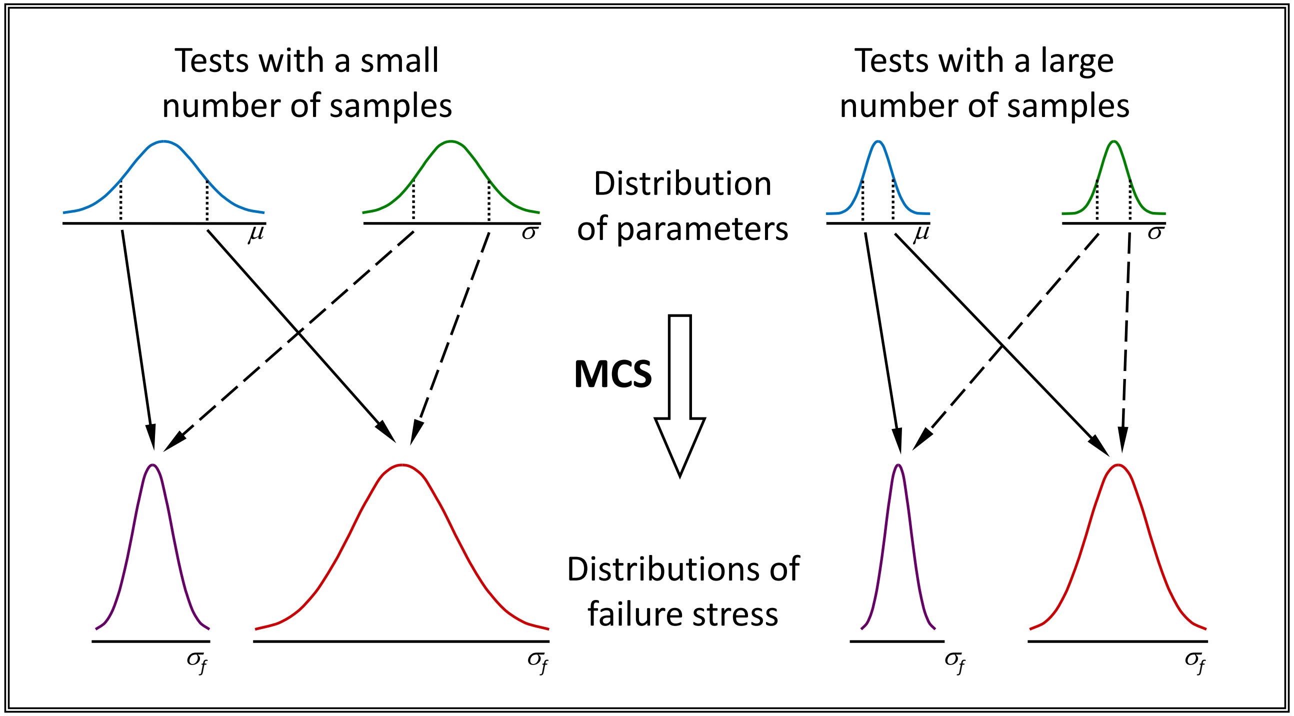






- Distribution in future of quartiles can be represented via normal distribution
- No clear effect of number of coupon tests

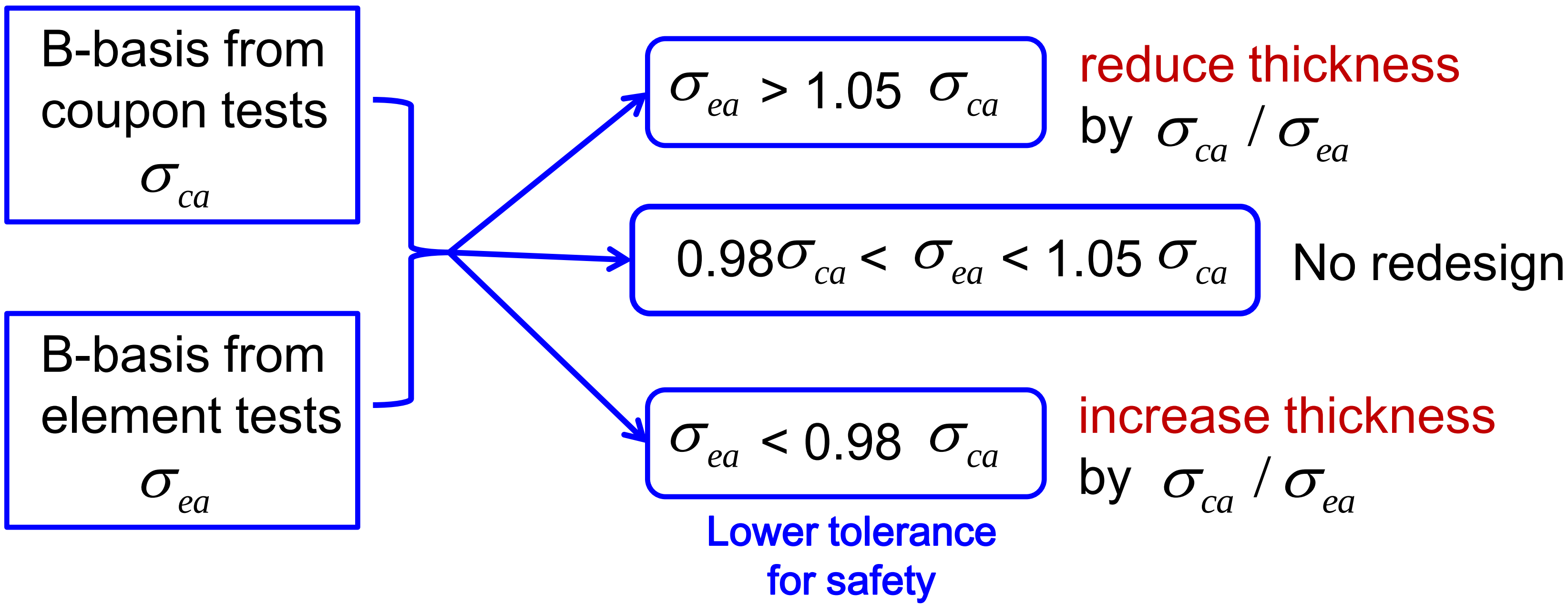
Simulation of future tests



	Mean values of the quartiles ($Q_{1\text{-}4}$)				Standard deviation of the quartiles ($Q_{1\text{-}4}$)			
	$\bar{Q}_1$	$\bar{Q}_2$	$\bar{Q}_3$	$\bar{Q}_4$	$std(Q_1)$	$std(Q_2)$	$std(Q_3)$	$std(Q_4)$
test1	0.901	0.970	1.051	1.146	0.092	0.098	0.106	0.115
test2	0.929	0.980	1.036	1.100	0.078	0.082	0.087	0.092
test3	0.940	0.982	1.028	1.078	0.073	0.076	0.080	0.084

Redesign based on element tests

If very large or very small failure stress values are obtained from the element tests, the company may want to increase or reduce the thicknesses of the elements.



Errors at the structural level

- Errors in structural strength predictions $\left(\bar{\sigma}_f\right)_{calc} = \left(1-e_f\right)\left(\bar{\sigma}_{ef}\right)_{calc}$
- Errors in load calculation $P_{calc} = (1+e_P)\,P_d$
- Error in stress calculation $\sigma_{calc} = (1+e_\sigma)\dfrac{P_{calc}}{w_{design}\,t}$ **assumed negligible!**
- Error in geometry $w_{built-av} = \left(1+e_w\right)w_{design}$ $t_{built-av} = \left(1+e_t\right)t_{design}$

Error factors	Distribution Type	Mean	Bounds
Error in load calculation, e_P	Uniform	0.0	± 10%
Error in width, e_w	Uniform	0.0	± 1%
Error in thickness, e_t	Uniform	0.0	± 3%
Error in failure prediction, e_f	Uniform	0.0	± 10%
Error in failure prediction, e_{sf}	Uniform	0.0	± 10%

Average built area

$$A_{built-av} = \left(1+e_{total}\right)\frac{S_F P_d}{k_f \sigma_{ea}}$$

$$e_{total} = \frac{\left(1+e_P\right)\left(1+e_t\right)\left(1+e_w\right)}{\left(1-e_f\right)} - 1$$

Design parameter (taken 0.95 here)

Variabilities

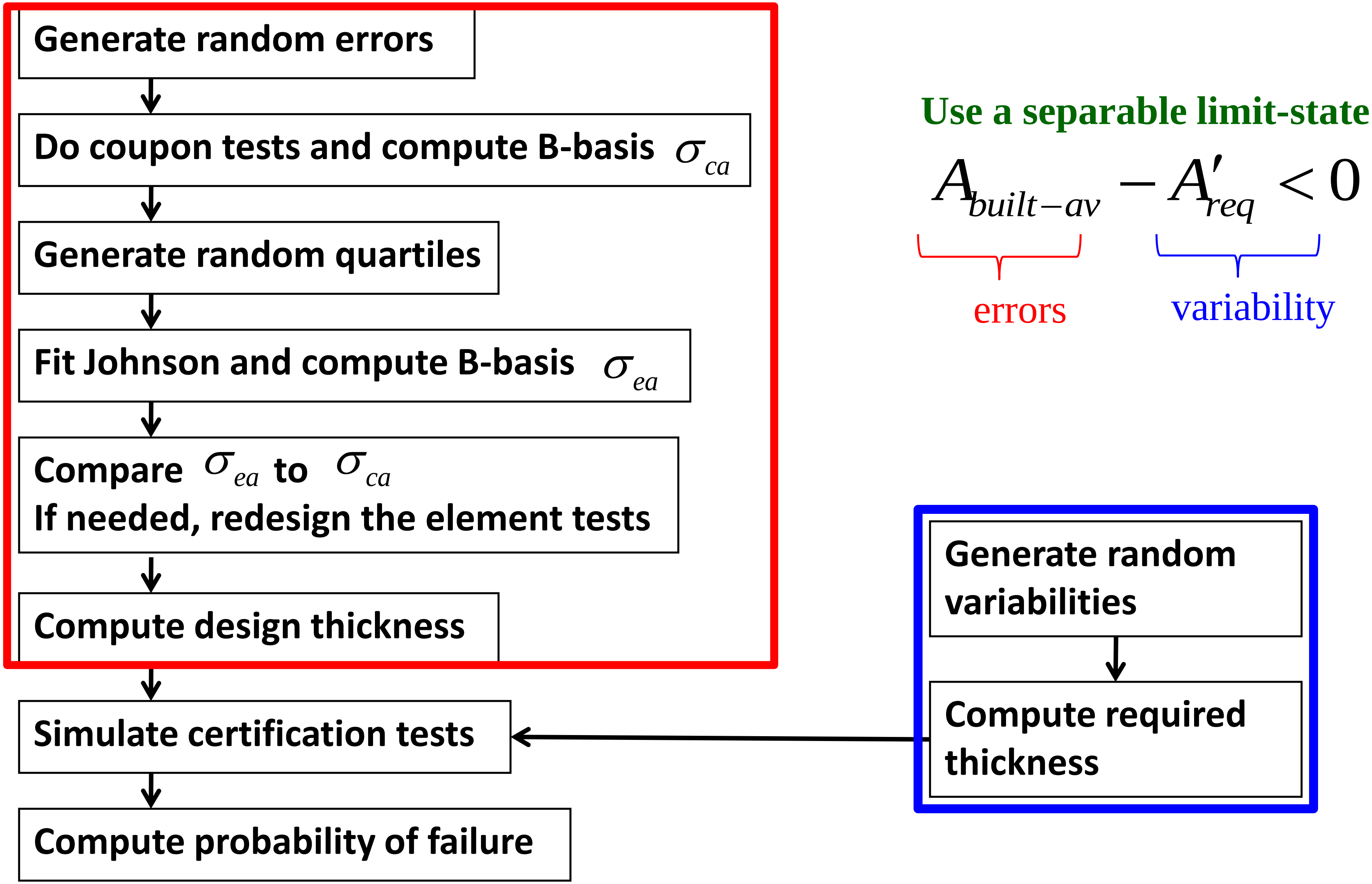
Variables	Distribution type	Mean	Scatter
Actual service load, P_{act}	Lognormal	$P_d = 2/3$	10% c.o.v.
Actual built width, $w_{built-var}$	Uniform	$w_{built-av}$	1% bounds
Actual built thickness, $t_{built-var}$	Uniform	$t_{built-av}$	3% bounds
Failure stress, σ_f	Normal	1.0	8% c.o.v.
v_w	Uniform	0	1% bounds
v_t	Uniform	0	3% bounds

- Variability in thickness $$t_{built-\mathrm{var}} = (1+v_t)\,t_{built-av}$$

Required area

$$\frac{A_{req}}{(1+v_t)(1+v_w)} \equiv A'_{req} \qquad \text{where} \quad A_{req} = P/\sigma_f$$

Reliability Calculation using Separable MCS



Results: Number of coupon tests, n_{coupon}

n_{coupon}	t_{mean}	t_{cov}	P_f	*PFCT*
30	1.268	0.116	1.02×10^{-4}	0.0576
50	1.253	0.114	1.27×10^{-4}	0.0654
80	1.245	0.113	1.44×10^{-4}	0.0711

PFCT: Probability of failing in certification tests

Current practice

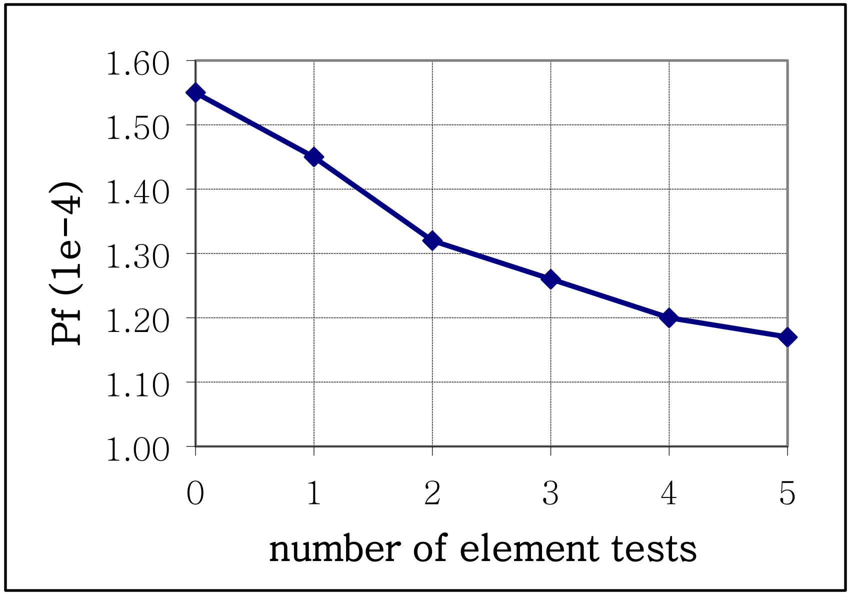
- As n_{coupon} reduces, weight increases, Pf and PFCT reduce. B-allowable is conservative!

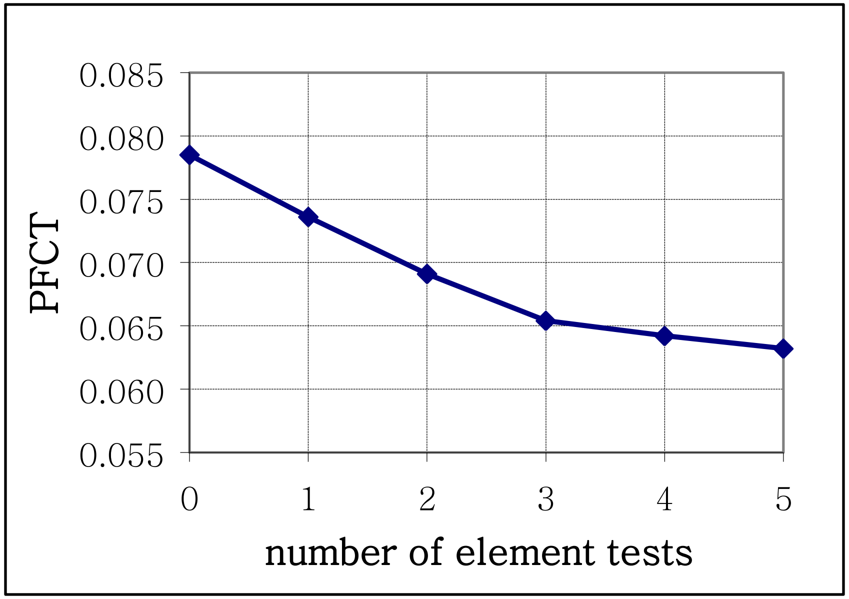
n_{coupon}	t_{mean}	P_f	*PFCT**
30	1.256	1.27×10^{-4}	0.0666
50	1.253	1.27×10^{-4}	0.0654
80	1.250	1.27×10^{-4}	0.0670

RBDO

- If we want to do away with 30 element tests only,
  - Need to put 0.5% extra weight

Number of element tests





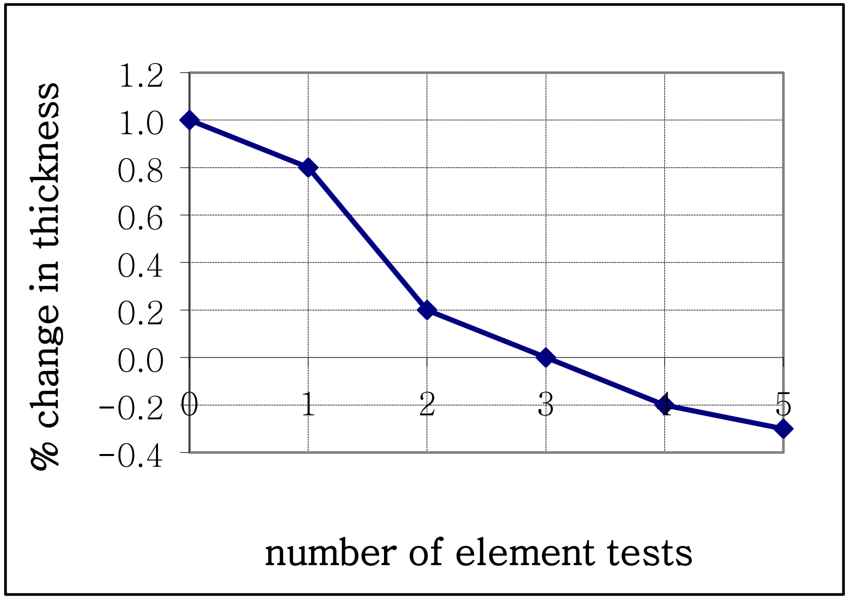
Current practice

- As the number of element tests is increased
  - Pf and PFCT reduces
  - Rate of the reduction diminishes

For the same weight!

What about RBDO?

n_e	t_{mean}	*% increase in thickness*	P_f	*PFCT*
0	1.266	1.0	1.27×10^{-4}	0.0653
1	1.263	0.8	1.27×10^{-4}	0.0656
2	1.256	0.2	1.27×10^{-4}	0.0668
3	1.253	---	1.27×10^{-4}	0.0654
4	1.250	-0.2	1.27×10^{-4}	0.0672
5	1.249	-0.3	1.27×10^{-4}	0.0672



- Element tests are more influential than coupon tests.
- Aircraft companies may reduce the number of coupon tests by moving to RBDO!

Certification test

	t_{mean}	t_{cov}	P_f
Certification	1.253	0.114	1.27×10^{-4}
No certification	1.244	0.119	2.31×10^{-4}
No certification with adjusted mean thickness	1.253	0.119	1.94×10^{-4}

- If certification test is not performed, the probability of failure is increased by **54%.**
- The certification test is an effective way of maintaining the reliability.

Concluding Remarks (1)

- As the number of coupon tests is increased
  - the mean allowable stress increases
  - so the mean thickness reduces.
- While the standard deviation of the thickness decreases, the probability of failure increases as does the probability of failing certification.
  - The FAA knockdown factor for compensating for small number of coupon tests is conservative.
- If we want to reduce the number of coupon tests for the same probability of failure,
  - need to put about 0.5% extra weight.

Concluding Remarks (2)

- If the number of element tests is increased
  - the probability of failure reduces
  - the rate of this reduction decreases
- If we want to dispense with element tests for same Pf
  - need to put about 1% extra weight.
- So, the number of element tests and coupon tests can be selected in a better way by moving to RBDO.
- If certification test is not performed, the probability of failure is increased by 54%, so the certification test is an effective way of maintaining the reliability.

Future Work: Simultaneous design of structure and tests

- Formulate an RBDO problem to

  Find k_f, n_c, n_e

  Min Weight (k_f, n_c, n_e)

  Such that Pf $(k_f, n_c, n_e) < Pf_{spec}$

- Will generate response surfaces for
  - Weight (k_f, n_c, n_e)
  - Pf (k_f, n_c, n_e)

Acknowledgements

- The authors gratefully acknowledge the support of the research by the NASA center and the U.S. Air Force, U.S. Air Force Research Laboratory, under award FA9550-07-1-0018 (Victor Giurgiutiu, Program Manager).