8th World Congress on Structural and Multidisciplinary Optimization
June 1 - 5, 2009, Lisbon, Portugal

Robust Shape and Topology Optimization of Compliant Mechanisms Considering Random Field Uncertainty

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1. Abstract
In this paper, we attempt to address the cutting-edge problem of robust shape and topology optimization (RSTO) of compliant mechanisms with consideration of random field uncertainty, such as material property. The proposed approach is based on the state-of-the art level set methods for shape and topology optimization and the latest research development in design under uncertainty. Conventional robust design, usually posed as a continuous optimization problem in finite dimensions, is extended to an infinite-dimensional shape and topology optimization problem and uncertainty is considered as a new dimension in addition to space and time. We illustrate that a level-set-based RSTO problem can be mathematically formulated by expressing the statistical moments of a response as functionals of geometric shapes and random field. To characterize the high-dimensional random-field material uncertainty with a reduced set of random variables, the Karhunen-Loeve expansion is employed, which is essentially a spectral representation of the random field using a reduced set of random variables and the eigenfunctions of its covariance function as expansion bases. Once the reduced set of random variables is identified, the univariate dimension-reduction (UDR) quadrature rule is employed for calculating statistical moments of the design response. The combination of the above techniques not only greatly reduces the computational cost in evaluating the statistical moments but also enables a semi-analytical approach that introduces the shape sensitivity of the statistical moments using shape sensitivity analysis. The application of our approach to compliant mechanism design shows that the proposed RSTO method can lead to designs with completely different topologies and superior robustness compared to their deterministic counterparts. Although the current contents of this paper are focused on Gauss-type material uncertainties, the proposed method is generic and can be easily extended to robust topology optimization subject to other types of uncertainties, such as Gauss/Non-Gauss type loading and geometric uncertainties.

2. Keywords: level set methods, robust design, topology optimization, shape optimization, uncertainty, material property, random field, compliant mechanisms

3. Introduction
Compliant mechanisms (CM) are elastic structures capable of transmitting energy and motion through structural deformation [1]. They are natural candidates for Micro-Electro-Mechanical Systems (MEMS) [2], high-precision machines [3] and surgical tools [4]. Structural optimization has played an important role in complaint mechanisms design, which aims to achieve cost-effective designs for a given amount of material. Based on the nature of an optimization task, structural optimization can be categorized into size, shape and topology optimizations, among which topology optimization is the most efficient way due to its capability of finding the optimal material distribution for a pre-specified design objective. The underlying idea of the existing topology optimization techniques [5] is to recast the design problem as an optimal material distribution problem so that the configuration of the design can fulfill the requirements measured quantitatively by an objective function. The differences among the aforementioned approaches lie in their representations and modeling schemes.

For designing compliant mechanisms, representative works in topology optimization include the ground structure based compliant mechanism optimization method by Kota [6], the homogenization-based methods, including the homogenization method and its variant SIMP (simple isotropic material with penalization) [7], and more recent work on the level set approach by Allaire et al. [8] and Wang et al.[9]. Despite large quantities of research conducted in topology optimization of compliant mechanisms with linear or nonlinear analysis, research in topology optimization of compliant mechanism under uncertainty has been limited. On the other hand, physical quantities such as loading and material properties at micro scale are random by nature but their impact on the overall design performance can not be ignored. To obtain robust and reliable CM designs, the input uncertainties should be characterized and its effects on the output performance should be quantified. Recent years have seen an emerging trend in topology optimization research to consider various uncertainties [10-12]. Olhoff et al. first integrated reliability analysis into SIMP method and introduced a new strategy called the reliability-based topology optimization (RBTO) [12], in which a probabilistic constraint is introduced but the objective is still treated as deterministic. Maute and Frangopol applied RBTO to MEMS-oriented compliant mechanism designs [10]. Kogiso et al. [11] proposed a sensitivity-based method for robust design of compliant mechanisms under loading uncertainties, where the variations of the output displacement with respect to the uncertainty of the direction of input forces are evaluated using the first-order derivative. Conti [13] is the first to combine the level set methods with stochastic programming techniques for structural optimization under loading uncertainties. Due to the computational challenges, in most of the current topology optimization work, material (or loading) uncertainty is simply treated either as a constant or as a random parameter varying uniformly across a spatial domain. For some engineering applications this is a valid assumption, but for many others, especially when designing compliant mechanisms at micro scale, the material property should be more realistically treated as a random field with spatial variation.

Our research objective in this work is to develop a mathematically rigorous and computationally viable approach that enables robust structural optimization of compliant mechanisms with the consideration of random-field uncertainty. Our special emphasis is on developing a robust shape and topology optimization (RSTO) method based on the level set approach, which provides a generic boundary representation model that is useful for both shape and topology optimizations. This paper is organized as follows: A brief review of robust optimization and fundamentals about level set methods for RSTO are presented in Section 4. After that, uncertainty characterization and propagation using the spectral method and the Gauss-type quadrature will be introduced in Section 5. In Section 6, the shape derivatives of the statistical moments are derived using the adjoint variable method. The numerical algorithm for RSTO together with the demonstration example is provided in Section 7. Conclusions and future works are discussed in the last section.

4. Level-Set Based Robust Shape and Topology Optimization (RSTO)

4.1. Robust design models

Conventional robust design, pioneered by Taguchi [14], refers to a class of methods for improving quality and reliability by designing a product or process so that it is robust (insensitive) against variations in uncontrollable noise variables [15, 16]. The robust design problem typically involves a nonlinear programming formulation [17, 18] in which the objective is to make suitable tradeoff between 'optimizing' the mean performance μ and minimizing the performance variance σ^2 (or the standard deviation σ), as shown in Figure 1.

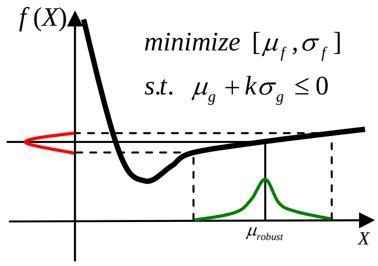


Figure 1: Robust design model [18].

The common robust design objective function balances between the mean and variance of the objective response through the choice of the constant k [19, 20]. Functions of the form $\mu + k\sigma$ also play a role when we have constraint responses that must satisfy certain conditions with specified probabilities. When the constraints relate to the failure of a product, the constraint evaluation is often referred to as reliability assessment [21-23]. A complete review of robust design optimization can be found in literatures [18, 24].

4.2. Level Set Methods for Shape and Topology Optimization

In the past two decades, level set methods have thrived to be powerful numerical tools with many applications in different fields [25, 26]. The advantage lies in their capability of precisely describing closed boundaries with dynamic variations, which enables easy 'capture' of the boundary on an Euler grid by solving a Hamilton-Jacobi partial differential equation [27]. Sethian and Wiengmann [28] first combined level set methods with the immersed interface methods for structural boundary design, where the former was used to represent the geometric boundary of the design and the latter was used for elastic analysis. Osher and Santosa [29] introduced the shape gradient of the objective functional into the level set model and established a link between the shape gradient and the velocity field. This work was further completed by Allaire et al. [8], who derived the shape sensitivity of compliance and geometric advantage by employing the adjoint variable method. Building upon the material derivative method, Wang et al. identified a meaningful link between the velocity field in the level set method and the general structural

sensitivity analysis [27]. The 'color level set' model proposed by Wang [30] made possible the topology optimization of multi-material compliant mechanisms in the level set framework [9].
As its name implies, level set method implicitly represents the boundary as the zero level set of a one-higher dimensional surface $\phi(x)$, which is called the level set function. In the level set model, the domain is defined as three parts according to the value of the level set function:

$$\begin{cases} \phi\left(x(t)\right) > 0 & : \quad x(t) \in D \setminus \Omega \\ \phi\left(x(t)\right) = 0 & : \quad x(t) \in \partial\Omega \\ \phi\left(x(t)\right) < 0 & : \quad x(t) \in \Omega \setminus \partial\Omega \end{cases}, \tag{1}$$

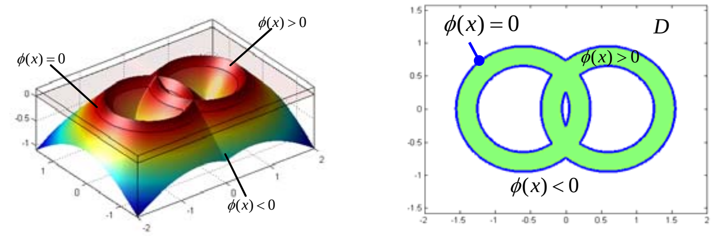
where D denotes the design domain; and $t \in R^{+}$ is time. The domain and a sketch of level set representation are shown in Figure 2. The greatest advantage of implicit representation lies in its ability of dealing with topological changes, such as splitting and merging of the boundary, in a natural manner.

By calculating the material derivative [31] of the equation $\phi\left(x\right) = 0$, we get the following equation:

$$\frac{\partial \phi}{\partial t} + \nabla\phi \cdot \mathbf{V}(x) = 0, \tag{2}$$

where $\mathbf{V}(x) = \dfrac{dx}{dt}$ is the velocity vector field. Considering $\mathbf{n} = \dfrac{\nabla\phi}{|\nabla\phi|}$ and $\mathbf{V} \cdot \nabla\phi = \left(\mathbf{V} \cdot \mathbf{n}\right)|\nabla\phi|$, we can write equation (2) as

$$\frac{\partial \phi}{\partial t} + V_n(x)|\nabla\phi| = 0. \tag{3}$$



(a) 3D level set function (b) corresponding 2D geometry

Figure 2: A 2d boundary embedded as the zero level set of a 3d level set function

These two Hamilton-Jacobi type partial differential equations (PDEs) are the well-known level set equations [25, 26, 32]. Based on the level set theory, the topology optimization problem is transformed into a problem of finding the steady-state solution of the Hamilton-Jacobi equation. To get a feasible steady-state solution of equations (2) and (3), an important issue is to find the velocity field. More details on calculating the shape derivative and identifying the velocity field in the RSTO problem will be provided in Section 5.

4.3. Setting an RSTO Problem for Compliant Mechanisms Design

In robust compliant mechanisms design, the objective is to optimize the performance subject to variations of random quantities. The performance index is the geometric advantage (GA) of the compliant mechanism, which is defined as the ratio between the output and input displacement. Uncertainty is introduced as a new dimension in addition to space and time [33], while the solution is sought in this extended space. We use z to denote the random quantities, and assume z being independent of the design variable - shape Ω. The design response (performance) under uncertainties can be correspondingly expressed as a functional $J(\Omega, u, z)$ of the random quantities z in addition to the geometric shape Ω and state variable u, that is

$$J(\Omega, u, z) = -\frac{\Delta_{out}\left(u(\Omega, z)\right)}{\Delta_{in}\left(u(\Omega, z)\right)}, \tag{4}$$

where Δ_{in} and Δ_{out} represent the displacements at the input port and the output port, as shown in Figure 3; the performance function $J(\Omega, u, z)$ is the geometric advantage.

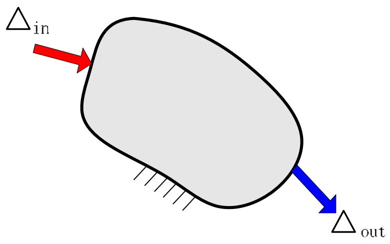


Figure 3: A schematic of a CM with input displacement Δ_{in} and output displacement Δ_{out} [34]

The random quantity considered here can have field variability to form a random field or random process but it can always be discretized into a finite number of random parameters, which will be further explained in Section 5.2. Thus equation (4) is general enough to cover random field or random process. The mean $\mu(J(\Omega,u,z))$ and standard derivation $\sigma(J(\Omega,u,z)$ of the performance index $J(\Omega,u,z)$ in equation (4) can be further expressed as follows

$$\begin{aligned}\mu(J(\Omega,u,z)) &= \int p(z)J(\Omega,u,z)dz = -\int p(z)\frac{\Delta_{out}\left(u(\Omega,z)\right)}{\Delta_{in}\left(u(\Omega,z)\right)}dz,\\ Var(J(\Omega,u,z)) &= \int p(z)\left[J(\Omega,u,z)-\mu\left(J(\Omega,u,z)\right)\right]^2 dz,\end{aligned} \tag{5}$$

where $p(z)$ is the joint probability density function (p.d.f.) of the random variables. In this way, a general RSTO problem is set as follows:

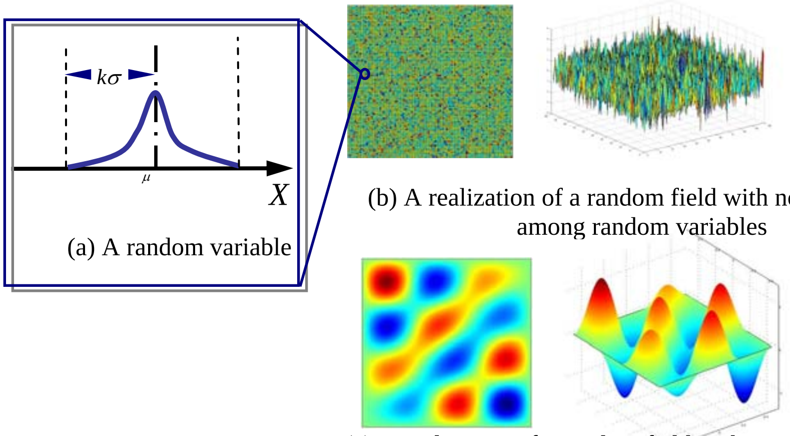
$$\begin{aligned}&\textit{Minimize}\\ &\qquad J^*(\Omega,u,z) = \mu(J(\Omega,u,z)) + k\sigma(J(\Omega,u,z)\\ &\textit{Subject to}:\\ &\qquad \textit{Volume constraint}\,|\Omega| = |\Omega|_{obj},\\ &\qquad \textit{Perimeter constraint on}\,|\partial\Omega|,\end{aligned} \tag{6}$$

together with the partial differential equations (PDEs) governing the physical system.

5. Uncertainty Quantification and Propagation in RSTO

5.1. Random Variables and Random Fields

An important issue in RSTO is how to model the input uncertainties. Following the probability theory, we can model uncertainties in structural optimization either as *random variables* or *random fields* [35]. The former can be considered as the constituting element of the latter, as shown in Figure 4. Random variables and random fields can be used to model different physical quantities. For example, when considering a concentrated random load, we can model its magnitude and direction as two random variables [36], either correlated or independent. But for problems with properties varying across the spatial domain, the physical quantities should be more realistically modeled as random fields [35]. A random variable often times can be conveniently characterized by the mean and variance of its distribution. To characterize a random field, a third factor needs to be taken into account, that is, the correlations (dependency) among the random variables in this random field. When there is no correlation or the correlation is very weak, the random field is more like the 'white noise' in signal analysis and Monte Carlo method can be used to model such a random field. When the correlation is strong, spectral methods [37] can be employed to quantify the uncertainty. In this work, we use a Gaussian random field with a relatively large correlation to describe the uncertain material field. To characterize the random-field material uncertainty with a reduced set of random variables, the Karhunen-Loeve (K-L) expansion approach is employed. To efficiently propagate uncertainty in a RSTO process, we propose to use the univariate dimension reduction (UDR) quadrature formula which is applicable to arbitrary probability distributions. The uncertainty modeled by a random field needs to be discretized into a finite number of random variables for practical manipulations. In this section, we first discuss the discretization of random fields using the K-L spectral representation and the propagation of uncertainty based on the UDR quadrature formula. These methods are further incorporated into the framework of level-set based RSTO.



(a) A random variable

(b) A realization of a random field with no correlation among random variables

(c) A realization of a random field with a strong correlation among random variables

Figure 4: A Random variable and a random field

5.2. Karhunen-Loeve Expansion of Random Field

The Karhunen-Loeve expansion [37] is a spectral approach to represent a random field using eigenfunctions of the random field's covariance function as expansion bases. Let $g(x,\omega)$ $: D\times\Theta\rightarrow\mathbb{R}$ be a random field defined over a spatial domain $\mathbf{D}$, which is a function of spatial coordinate x. Here $\omega\in\Theta$ denotes an element of the sample space and is used to indicate that the involved quantity is random. $g(x,\omega)$ can be represented by the K-L expansion as follows:

$$g(x,\omega)=\bar{g}(x)+\sum_{i=1}^{\infty}\sqrt{\lambda_i}g_i(x)\xi_i(\omega), \tag{7}$$

where $\bar{g}(x)$ is the mean function. λ_i and $g_i(x)$ are the i$^{\text{th}}$ eigenvalue and eigenfunction obtained from the following integral equation:

$$\int_{\mathbf{D}}C(x_1,x_2)g_i(x_1)dx_1=\lambda_i g_i(x_2), \tag{8}$$

where $C(x_1,x_2)$ is the spatial covariance function of the random field $g(x,\omega)$. The random field variables, $\xi_i(\omega)$ in Eqn. (7) are orthogonal random variables with zero mean and unit variance. That is,

$$E(\xi_i(\omega))=0 \text{ and } E(\xi_i(\omega)\xi_j(\omega))=\delta_{ij}\,. \tag{9}$$

The orthogonality of $\xi_i(\omega)$ is a unique feature of the K-L expansion. $\xi_i(\omega)$ can be calculated as:

$$\xi_i(\omega)=\frac{1}{\sqrt{\lambda_i}}\int_D\left(g(x,\omega)-\bar{g}(x)\right)g_i(x)dx \tag{10}$$

The second order statistics of $\xi_i(\omega)$ in Eqn. (9) can be derived from Eqn. (10). Based on sampling and spatial integration at the right side of Eqn. (10), samples of $\xi_i(\omega)$ can be generated to infer the distribution of the random field variable, $\xi_i(\omega)$. The K-L expansion is the optimal among finite representations using orthogonormal bases in the sense that the mean square error caused by a truncation of the expansion is minimized [37].

5.3. Univariate Dimension Reduction (UDR) quadrature method for statistical moments calculation

Multivariate quadrature formulas for multiple random variables can be built in many different ways [38] from one dimensional quadrature formulas [59]. With the univariate dimension reduction (UDR) method [39], the multivariate function $g(\mathbf{X})$ is approximated by a sum of univariate functions which depend on only one variable with the other variables fixed to their mean values. Let the univariate functions denoted by $g_{\_i}$, and then $g(\mathbf{X})$ is approximated as follows:

$$g(\mathbf{X})\simeq\hat{g}(\mathbf{X})=\sum_{i=1}^{n}g(\mu_1,\cdots,X_i,\cdots,\mu_n)-(n-1)g(\mu_1,\cdots,\mu_n)=\sum_{i=1}^{n}g_{\_i}(X_i)-(n-1)g(\boldsymbol{\mu}_{\mathbf{X}}). \tag{11}$$

Here independence of X_i is assumed and it is known that the error of this approximation is mainly contributed by the interaction effects among variables [40]. Since X_i are mutually independent, $g_{\_i}(X_i)$ are also independent with each other and the statistical moments of $\hat{g}(\mathbf{X})$ can be approximated conveniently from moments of $g_{\_i}(X_i)$, as follows [41]:

$$\text{(Mean)} \quad \mu_{\hat{g}} = \sum_{i=1}^{n} \mu_{g_{\_i}} - (n-1)\, g(\boldsymbol{\mu}_{\mathbf{X}}), \tag{12}$$

$$\text{(STD)} \quad \sigma_{\hat{g}}^2 = \sum_{i=1}^{n} \sigma_{g_{\_i}}^2 .$$

The moments of univariate functions are calculated using one dimensional Gauss-type quadrature formula. The number of $g(\mathbf{x})$ evaluations for this calculation is $m_1 + \cdots + m_n + 1$ where m_i is the number of nodes used for the calculation of moments of $g_{\_i}$. It needs to be noted that though UDR method might be the most efficient, the method might lose accuracy when there exist strong interactions between random variables [42].

6. Shape Derivatives of Statistical Moments

In optimization search of solving RSTO problems, to minimize the objective functional formulated in equation (6), we need to quantify the change of the objective functional $J^*(\Omega, u, \omega)$ with respect to a small variation of the shape Ω (design), which can provide us with necessary information for updating the current design. This process is called shape sensitivity analysis and the result is called shape derivative [43]. In this section, a semi-analytical shape sensitivity analysis approach is presented. The mean and variance of a response are first numerically discretized using the multivariate Gauss-type quadrature approach discussed in Section 3.3. From an optimization point of view, the multivariate Gauss-type quadrature essentially transforms the RSTO problem into a weighted summation of a series of deterministic topology optimization subproblems. The shape sensitivity of each subproblem is then derived using the adjoint variable method and calculus of variation.

Following this approach, equations (6) can be approximated as follows by using the UDR formula in equations (11) and (12).

$$\begin{aligned} \mu_J &= \sum_{i=1}^{n} \mu_{J_{\_i}} - (n-1)\, g(\boldsymbol{\mu}_{\boldsymbol{\omega}}) \\ \sigma_J^2 &= \sum_{i=1}^{n} \sigma_{J_{\_i}}^2 \end{aligned} \tag{13}$$

where $J_{\_i}$ is the objective function value calculated with ω_i as a realization of the random variables and other random variables fixed to their mean values; n is the number of quadrature points. We address the general problem using the variational method and the techniques proposed in [8, 44]. With the assumption that the random variables are independent of the design variables Ω, the shape derivatives of the mean and variance of the performance function $J(\Omega, u, \omega)$ are expressed as follows:

$$D_\Omega[\mu_J] = \sum_{i=1}^{n} D_\Omega\left[\mu_{J_{\_i}}\right] - (n-1) D_\Omega\left[g(\boldsymbol{\mu}_{\boldsymbol{\omega}})\right] \tag{14}$$

Similarly, the shape derivative of the variance can be expressed as:

$$D_\Omega\left[\sigma_J^2\right] = \sum_{i=1}^{n} D_\Omega\left[\sigma_{J_{\_i}}^2\right] \tag{15}$$

The final shape derivative of the objective functional $J^*(\Omega, u, \omega)$ is

$$D_\Omega\left[J^*(\Omega, u, \omega)\right] = D_\Omega\left[\mu(J(\Omega, u, \omega))\right] + k D_\Omega\left[\sigma(J(\Omega, u, \omega))\right] = \sum_{i=1}^{n} D_\Omega\left[\mu_{J_{\_i}}\right] + k \sum_{i=1}^{n} D_\Omega\left[\sigma_{J_{\_i}}^2\right] \tag{16}$$

With certain mathematical manipulations, the shape sensitivity of $D_\Omega\left[J^*(\Omega, u, \omega)\right]$ can be transformed into a weighted summation of the shape sensitivities of a series of deterministic scenarios denoted as $D_\Omega\left[J(\Omega, u, \omega_i)\right]$. $D_\Omega\left[J(\Omega, u, \omega_i)\right]$ reveals the underlying relations between the design variable shape Ω and the objective functional $J(\Omega, u, \omega_i)$ under a specified load scenario with the random parameter ω_i. For more details of sensitivity analysis for level-set based topology optimization, please be referred to our previous papers [8, 9, 27, 34]. In terms of a linear elastic problem, the final form of $D_\Omega\left[J(\Omega, u, \omega_i)\right]$ can be formulated as follows [9, 34]:

$$D_\Omega\left[J(\Omega,u,\omega_i)\right]=\int_\Gamma\left\{\frac{\partial J}{\partial u_{1i}}E_{ijkl}\varepsilon_{ij}(u_1)\varepsilon_{kl}(u_1)+\left(\frac{\partial J}{\partial u_{1o}}+\frac{\partial J}{\partial u_{2i}}\right)E_{ijkl}\varepsilon_{ij}(u_1)\varepsilon_{kl}(u_2)+\frac{\partial J}{\partial u_{2o}}E_{ijkl}\varepsilon_{ij}(u_2)\varepsilon_{kl}(u_2)\right\}V_n ds \tag{17}$$

where u_1 and u_2 denote the displacement field caused by the unit force f_{in} and f_{out} respectively; u_{1i} denotes the displacement at the input port in the same direction as f_{in} ; u_{1o} denotes the displacement at the output port in the same direction as the output force [9, 34]. With equation (17), $D_\Omega\left[\mu_{J\_i}\right]$ and $D_\Omega\left[\sigma^2_{J\_i}\right]$ can be easily calculated using one dimensional Gauss-type quadrature formula. Substituting $D_\Omega\left[\mu_{J\_i}\right]$ and $D_\Omega\left[\sigma^2_{J\_i}\right]$ into equation (16), we obtain the shape gradient of the objective functional.

7. RSTO Algorithm and Demonstration Examples

7.1. Numerical Algorithm

The algorithm for RSTO is shown in Figure 4. After setting the initial design and boundary conditions, the K-L expansion method introduced in Section 5 is first employed to reduce the dimensionality for representing the uncertainties in loading and material. For the reduced set of random variables, the locations and weights of nodes are determined next based on the Gauss-type quadrature for calculating the mean and variance of the performance function. The shape sensitivity is then calculated at each integration node. Therefore, the computational cost is proportional to the number of nodes. The velocity field is set using the steepest descent method and the geometry is updated via Hamilton-Jacobi equation. This loop will iterate until the convergence criterion is satisfied.

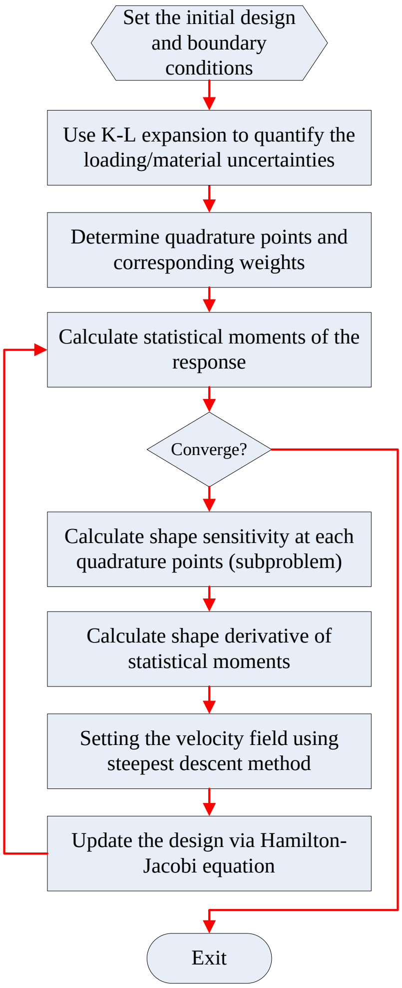


Figure 5: Flowchart of the algorithm

7.2. Demonstration Example -A 3D Micro-Gripper under a Random Material Field

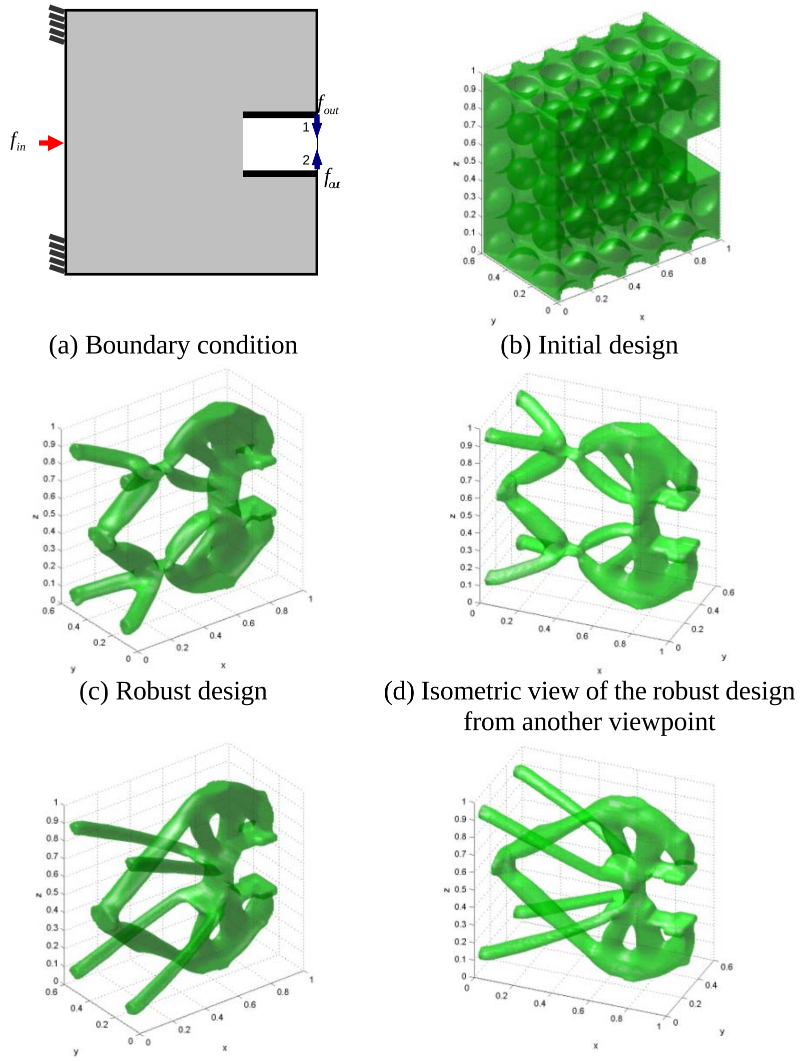
A 3D micro-gripper is used as a demonstrative example, which is subject to a spatially-varying material property field across the design domain. The boundary condition of the problem is shown in Figure 6(a): the four corners of the left side are fixed with a horizontal force applied at the center of the left side; two vertical output displacements are expected at ports 1 and 2. The objective function is the geometric advantage (GA), which is defined as the ratio of the output displacement over the input displacement. For more details about the setting of a compliant mechanism optimization problem, please be referred to papers [9, 34, 45]. The dimensions of the design domain are 1-by-0.6-by-1. The material property field (Young's Modulus) is assumed to take a normal distribution with mean equal to 1 and standard deviation 0.3. An exponential function is employed to describe the correlation between any two spatial points in the random field as follows:

$$C=\exp\left(-\frac{\left\|X_1-X_2\right\|}{d}\right). \tag{18}$$

Here $\left\|X_1-X_2\right\|$ is the Euclidean distance between the two points and d is the correlation length which is set to be 0.5 in this example. Due to the symmetry, only the upper part of the design is analyzed in the optimization process. The level set function is evolved on a 101-by-61-by-51 Euler grid, and the design domain is discretized using about 5000 finite elements for elastic analysis. Due to the strong correlation, three eigenvectors are used with K-L expansion in uncertainty quantification and the random material field can be quantified as follows:

$$g(x,\omega) = \overline{g}(x) + \sum_{i=1}^{3} \sqrt{\lambda_i} g_i(x) \xi_i(\omega) \tag{19}$$

Three quadrature nodes are used in each eigenvector direction based on the UDR approach to uncertainty propagation. The final design of RSTO is shown in Figures 6 (c) and (d). The corresponding deterministic topology optimization (DTO) result is shown in Figures 6 (e) and (f), where the Young's Modulus is a constant. The performances of robust and deterministic designs are listed in Table 3. We apply two different material fields to both the robust and the deterministic designs. A clear observation is that robustness is achieved at the cost of sacrificing performances. However, according to Table 3, the geometric advantage of the robust design is more stable (varying from -0.065 to -0.059) under different material fields, while the geometric advantage of deterministic design degenerates when the material property field varies (varying from -0.070 to -0.055). When comparing the geometries of the robust and deterministic designs, we find the concepts obtained are quite different: the robust design consists of shorter but obviously thicker bars than its deterministic counterpart. It needs to be pointed out that the 'robustness' here is only in a mathematical sense (a smaller variation of geometric advantage). Since the stress constraint is not considered in current formulations, the robust design achieved in this example possesses more de facto hinges, while the deterministic design possesses more distributed compliance. This makes the stress concentration easier to occur in the robust design than the deterministic design; a pitfall can be addressed by adding stress constraints into the problem formulation.



(a) Boundary condition

(b) Initial design

(c) Robust design

(d) Isometric view of the robust design from another viewpoint

(e) Deterministic design

(f) Isometric view of the deterministic design from another viewpoint

Figure 6: Robust (c-d) vs. Deterministic (e-f) optimization of a 3d micro-gripper under a random material field.

Table 3: Geometric Advantages of Robust and Deterministic Designs under Different Material Property Fields

	Parameters of Material Fields	Volume Ratio	Robust Design	Deterministic Design
Material Field 1	$E = 1$	0.090	-0.065	-0.070
Material Field 2	$\mu_E = 1$, $\sigma_E = 0.3$, $d = 0.5$	0.098	-0.059	-0.055

5. Conclusions

We integrated robust design with level set methods to implement robust shape and topology optimization with demonstration to robust compliant mechanism design subject to random material field. The Karhunen-Loeve expansion is employed to characterize random-filed uncertainty, which is essentially a spectral representation of the random field using a reduced set of random variables and the eigenfunctions of its covariance function as expansion bases. Once the reduced set of random variables is identified, the univariate dimension-reduction (UDR) quadrature rule is then employed for calculating statistical moments of the design response. The combination of the above techniques not only provides a computationally viable approach in evaluating the statistical moments, which otherwise would be computationally formidable, but also enables a semi-analytical approach that introduces the shape sensitivity of the statistical moments using the adjoint variable method and calculus of variation. The shape derivative is seamlessly integrated with a level-set-based topology optimization framework via the steepest descent method. The benchmark example shows that the results from RSTO may be quite different from that of the deterministic topology optimization and the RSTO designs are more robust than deterministic designs under uncertainty. Throughout our research, we also found that uncertainty is not the only factor that has an impact on the topology of the final design; the interaction between the boundary condition and the uncertainties determines the topology of the final design to a large extent (keeping other conditions fixed). The impact of such interactions still needs further investigations in our future research.

6. Acknowledgements

The grant support (CMMI – 0522662) from National Science Foundation (NSF) and the support from the Center for Advanced Vehicular Systems at Mississippi State University via Department of Energy Contract No: DE-AC05-00OR22725 are greatly acknowledged.

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