Robust Shape and Topology Optimization of Compliant Mechanisms Considering Random Field Uncertainty

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Outline

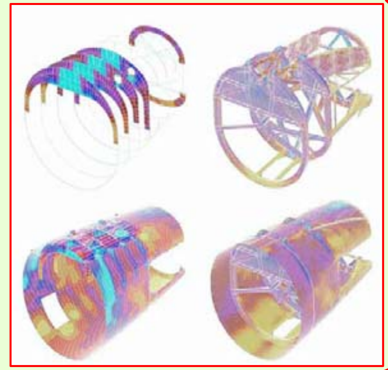
1. Research Objective;
2. Background Introduction;
3. Level-set Based Robust Shape & Topology Optimization;
4. Karhunen-Loeve Expansion for Quantification of Random-Field Uncertainties;
5. Univariate Dimension Reduction (UDR) Method for Uncertainty Propagation;
6. Shape Derivatives of Statistical Moments;
7. Numerical Algorithm & Benchmark Examples;
8. Conclusions & Future Work.

Research Objective

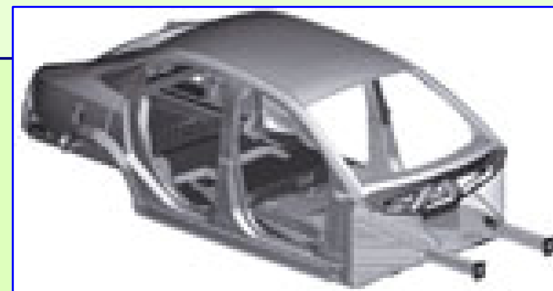
- Objective: to develop a mathematically rigorous and computationally viable approach that enables robust structural optimization of compliant mechanisms with the consideration of random-field uncertainty.

Robust Topology Optimization

Topology Optimization generates the optimal geometrical configuration of a mechanical design without needing a priori knowledge of the geometry.



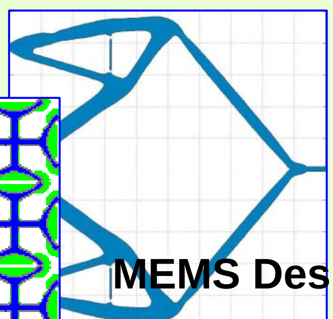
Aircraft Structure Design



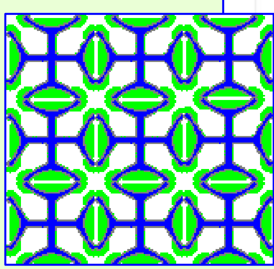
Light Vehicle Frame Design

Macro World

Micro World



MEMS Design



Microstructure of composite material

Most of the state-of-the-art work in topology optimization is deterministic, focusing on performance-satisfied design without considering robustness or reliability.

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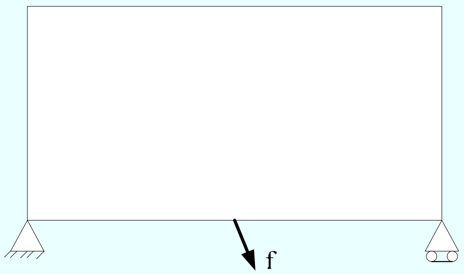
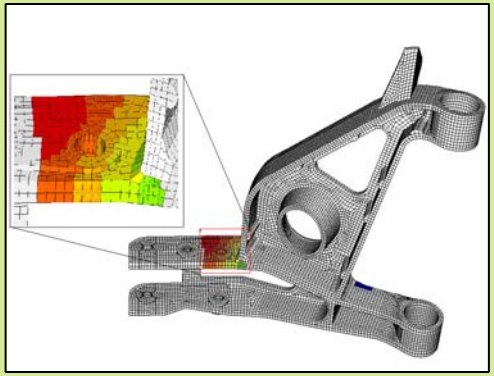
Sources of Uncertainties in Topology Optimization

Sources of uncertainties

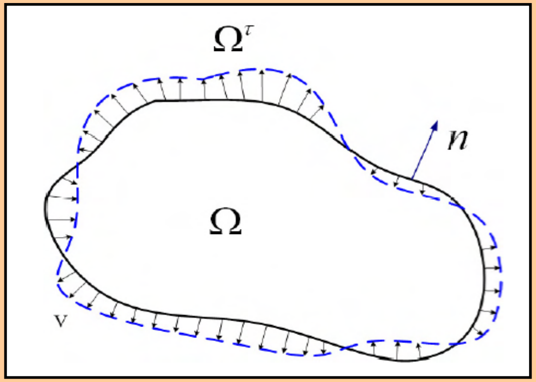
- Loading – varying working conditions
- Material - by nature
- Geometry - manufacturing



Loading uncertainty



Material uncertainty



- *Geometric uncertainty*

Research in Topology Optimization under Uncertainty

Topology Optimization

- Infinite-dimensional optimization problem
- Complex sensitivity analysis due to complex physics modeling
- Often times an ill-posed problem

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Design under Uncertainty

- Uncertainty quantification
- Uncertainty propagation

→

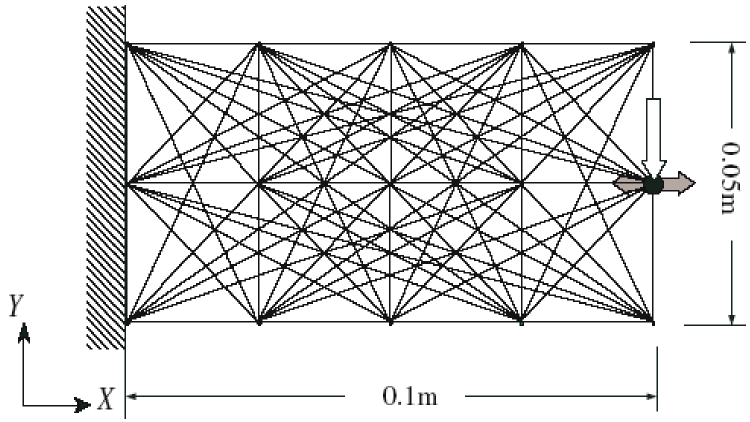
Topology Optimization under Uncertainty

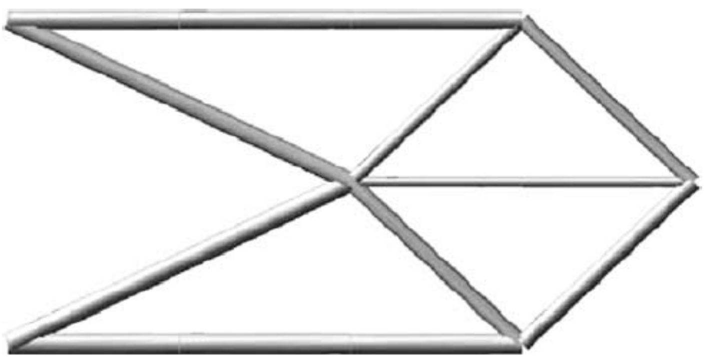
- Modeling and propagation of high-dimensional uncertainties
- Sensitivity analysis considering uncertainty

Topology Optimization under Uncertainty: State of The Art

- Truss-based topology optimization under uncertainty

Reliability-based design (Christiansen, Patriksson et al. 2001; Mogami, Nishiwaki et al. 2006)
Robust design (Seepersad, Allen et al. 2006)

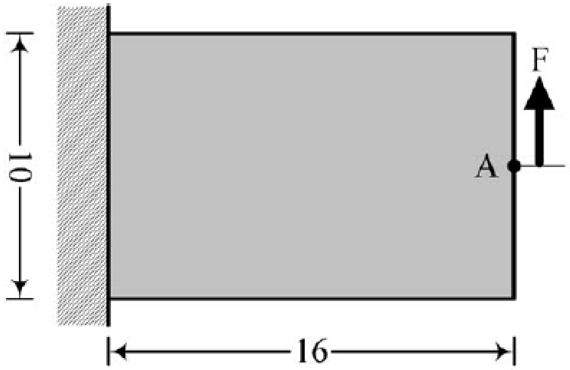




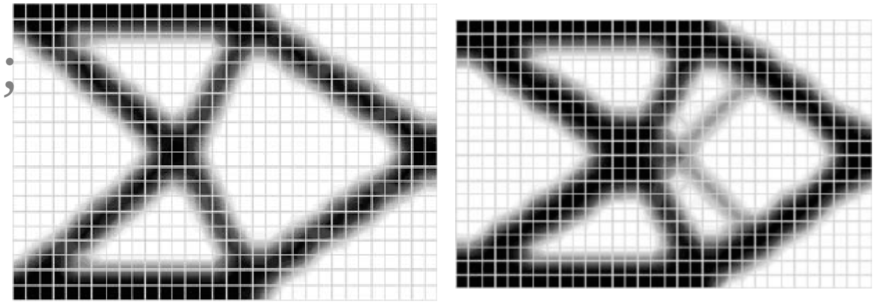
Mogami, K., S. Nishiwaki, et al. (2006). "Reliability-based structural optimization of frame structures for multiple failure criteria using topology optimization techniques " Structural and Multidisciplinary Optimization **32**(4): 299-311.

Topology Optimization under Uncertainty: State of The Art

- Homogenization/SIMP-based topology optimization under uncertainty



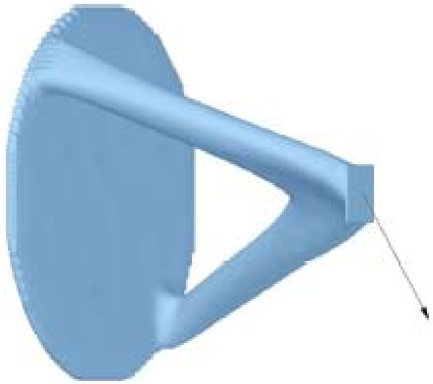
Reliability-based design (Kharmanda and Olhoff 2002; Maute and Frangopol 2003; Jung and Cho 2004; Kharmanda, Olhoff et al. 2004)



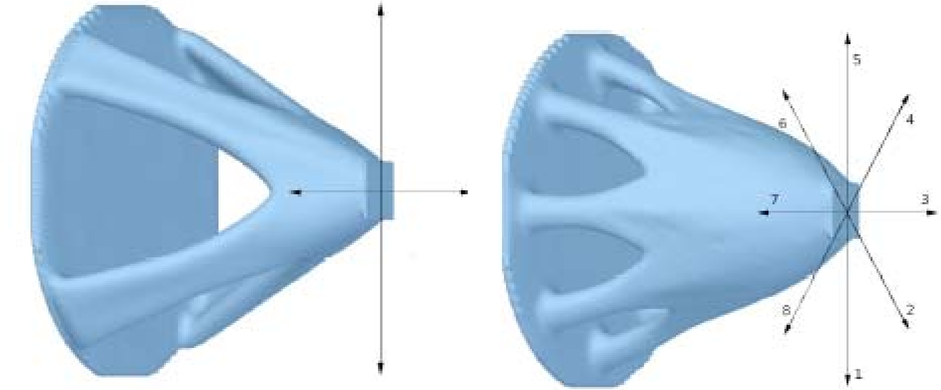
Deterministic approach; reliability-based approach (PMA)

Jung, H.-S. and S. Cho (2004). "Reliability-based topology optimization of geometrically nonlinear structures with loading and material uncertainties." Finite Elements in Analysis and Design **41** (3): 311-331.

- Level-set-based topology optimization under uncertainty

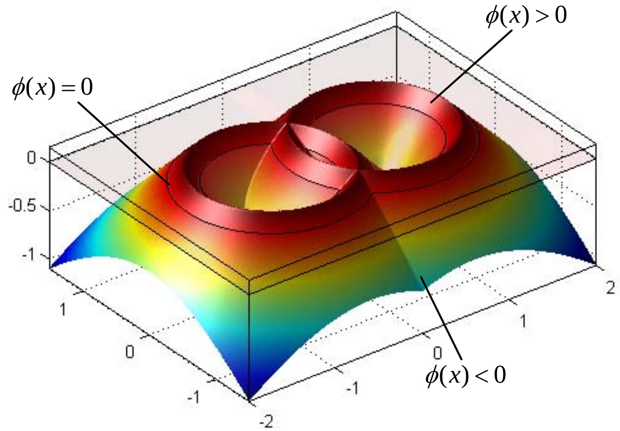


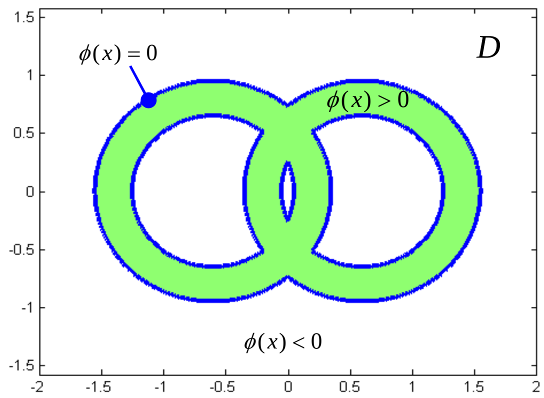
Stochastic programming (Conti, Held et al. 2008)



CONTI, S., H. HELD, et al. (2008). SHAPE OPTIMIZATION UNDER UNCERTAINTY - A STOCHASTIC PROGRAMMING PERSPECTIVE.

Level Set Methods





$$\begin{cases} \phi(x) > 0 & \forall x \in \Omega \setminus \partial\Omega \\ \phi(x) = 0 & \forall x \in \partial\Omega \\ \phi(x) < 0 & \forall x \in D \setminus \Omega \end{cases}$$

- **Implicit representation**
- **Benefits**
  - Precise representation of boundaries
  - Can capture any shape or topology
  - No chess-board patterns
- **Applications**
  - Computational fluid dynamics (Osher and Sethian 1988)
  - Image processing (Osher and Fedkiw 2003)
  - Shape modeling (Malladi, Sethian et al. 1995)
  - Structure optimization (Sethian, 2002, Wang 2003, 2004; Allaire, 2002, 2004)

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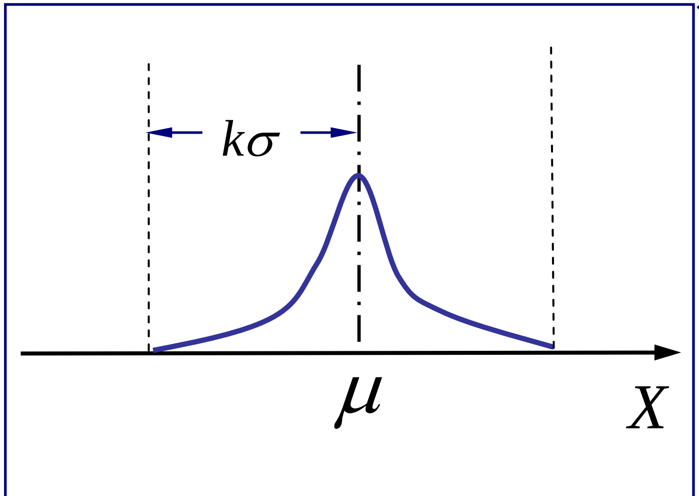
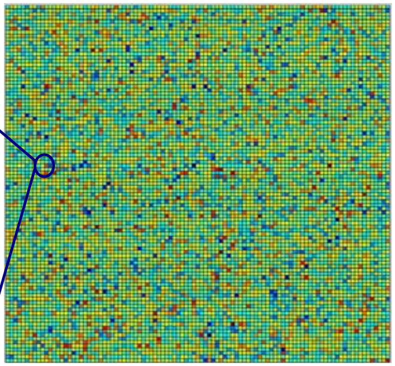
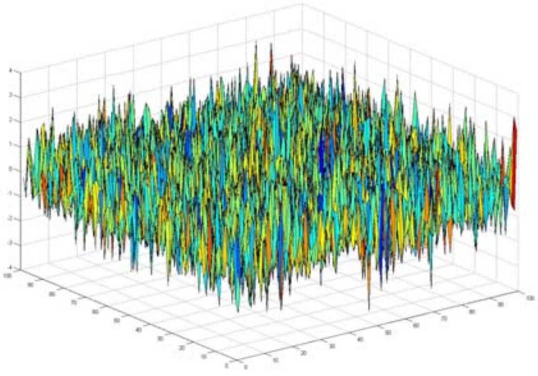
Random Variable and Random Field

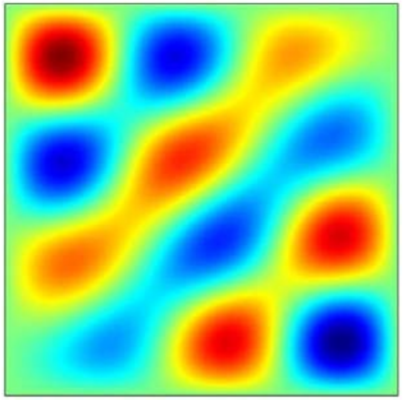


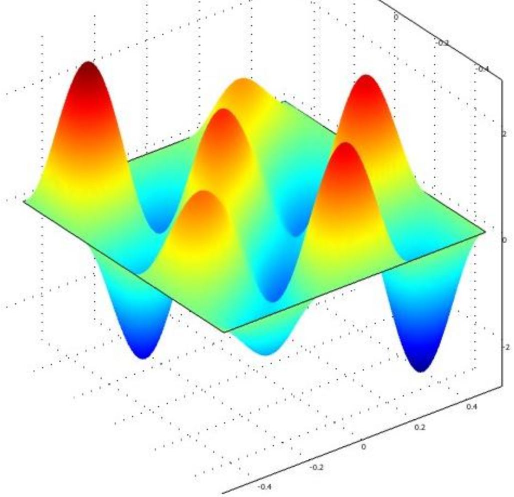
A random variable





A realization of a random field with <u>no correlation</u> between random variables





A realization of a random field with a <u>strong correlation</u> between random variables

Material properties can be more realistically modeled as a random field.

Uncertainty Representation - Truncated Karhunen-Loeve (K-L) Expansion of Random Field

- **Karhunen-Loeve Expansion**
- A spectral approach to represent a random field using eigenfunctions of the random field's covariance function as expansion bases.

Random Field

$$g(x;\theta) = \bar{g}(x) + \sum_{i=1}^{\infty} \sqrt{\lambda_i} g_i(x) \xi_i(\theta)$$

i^{th} eigenvalue i^{th} eigenvector

ξ: orthogonal random parameters

$\bar{g}$: mean function

x **- spatial coordinate**

θ- random parameter

Significance check

Select M when s is close to 1

$$s = \frac{\sum_{i=1}^{M} \lambda_i}{\sum_{i=1}^{N} \lambda_i}$$

Truncated K-L Expansion

$$g(x;\theta) \approx \bar{g}(x) + \sum_{i=1}^{M} \sqrt{\lambda_i} \cdot g_i(x) \xi_i(\theta)$$

Ghanem and Spanos 1991; Haldar and Mahadevan 2000; Ghanem and Doostan 2006

Uncertainty Propagation: Univariate Dimension Reduction (UDR) Method

- The multivariate function $g(\mathbf{X})$ is approximated by a sum of univariate functions which depend on only one variable with the other variables fixed to their mean values.

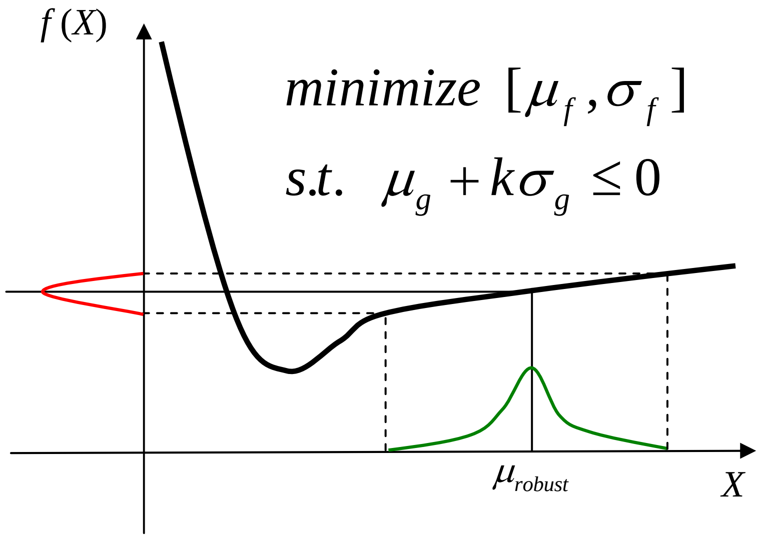
$$g(\mathbf{X}) \simeq \hat{g}(\mathbf{X}) = \sum_{i=1}^{n} g(\mu_1, \cdots, X_i, \cdots, \mu_n) - (n-1)\, g(\mu_1, \cdots, \mu_n)$$

$$= \sum_{i=1}^{n} g_{\_i}(X_i) - (n-1)\, g(\boldsymbol{\mu}_{\mathbf{X}})$$

$$\mu_{\hat{g}} = \sum_{i=1}^{n} \mu_{g_{\_i}} - (n-1)\, g(\boldsymbol{\mu}_{\mathbf{X}})$$

$$\sigma_{\hat{g}}^2 = \sum_{i=1}^{n} \sigma_{g_{\_i}}^2$$

Formulations for RSTO of Compliant Mechanisms



Robust Design Model (Chen, 1996)

Minimize

$$J^*(\Omega,u,z)=\mu(J(\Omega,u,z))+c\sigma(J(\Omega,u,z))$$

Subject to:

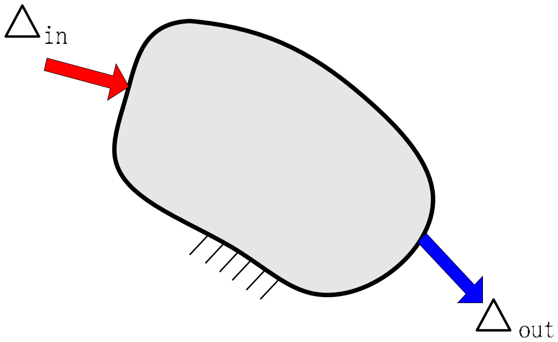
$$\text{Volume constraint } |\Omega| = |\Omega|_{obj},$$

$$\text{Perimeter constraint on } |\partial\Omega|,$$

$$-div\sigma(u)=f \quad in\ \Omega,$$

$$u=0 \quad on\ \Gamma_D,$$

$$\sigma(u)\cdot n=g \quad on\ \Gamma_N.$$



$$J(\Omega,u,z)=-\frac{\Delta_{out}\left(u(\Omega,z)\right)}{\Delta_{in}\left(u(\Omega,z)\right)}$$

$$\mu(J(\Omega,u,z))=\int p(z)J(\Omega,u,z)dz=-\int p(z)\frac{\Delta_{out}\left(u(\Omega,z)\right)}{\Delta_{in}\left(u(\Omega,z)\right)}dz$$

$$Var(J(\Omega,u,z))=\int p(z)\left[J(\Omega,u,z)-\mu\left(J(\Omega,u,z)\right)\right]^2 dz$$

(Chen, 2005)

Shape Sensitivity Analysis

Expand the functions of mean and variance using UDR in an additive format

$$D_\Omega[\mu_J] = \sum_{i=1}^{n} D_\Omega\left[\mu_{J\_i}\right] - (n-1) D_\Omega\left[g\left(\boldsymbol{\mu}_\omega\right)\right]$$

$$D_\Omega\left[\sigma_J^2\right] = \sum_{i=1}^{n} D_\Omega\left[\sigma_{J\_i}^2\right]$$

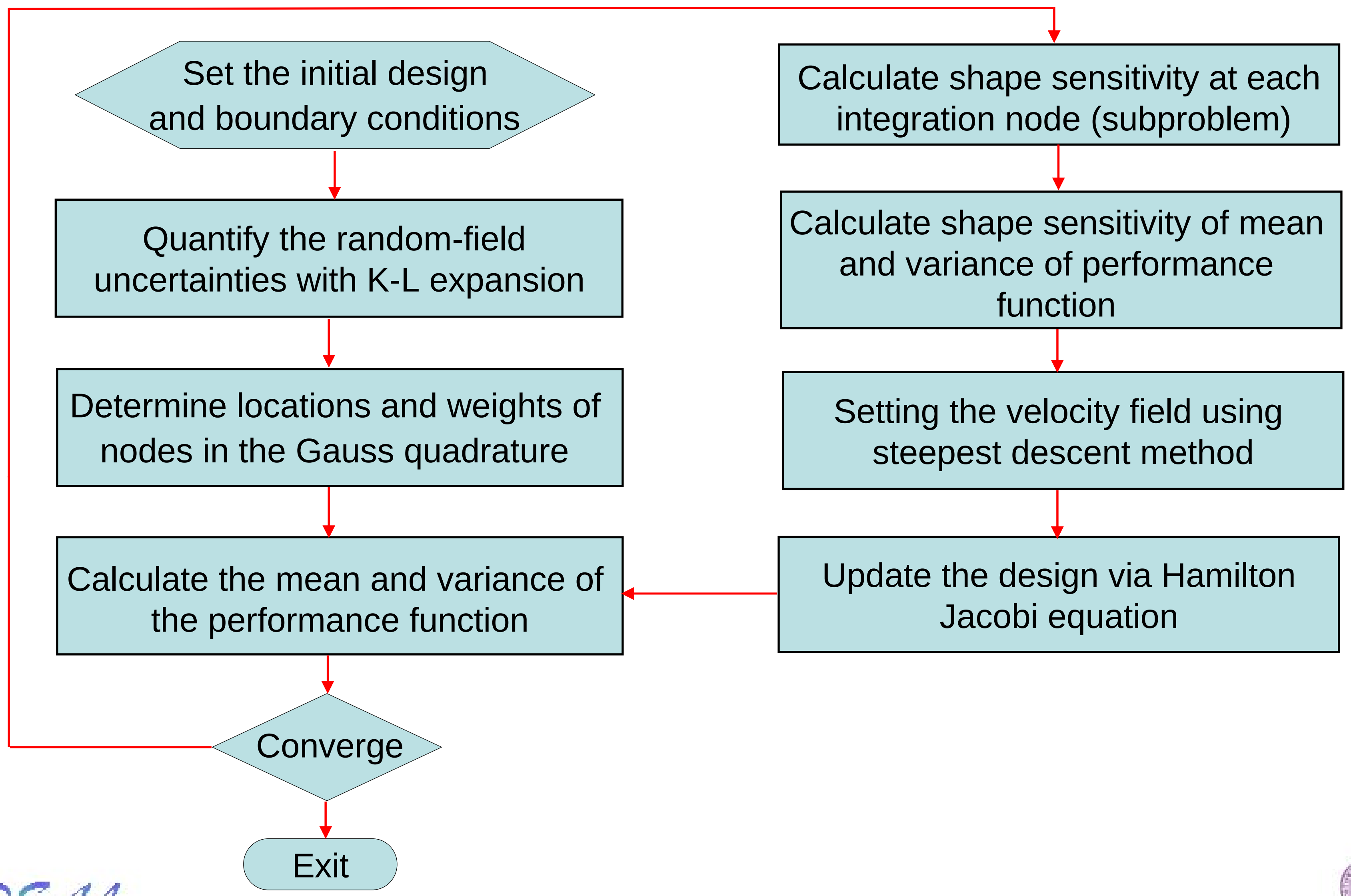
Using adjoint method and shape sensitivity analysis (Sokolowski, 1992), we can obtain

$$D_\Omega\left[J^*(\Omega,u,\omega)\right] = D_\Omega\left[\mu(J(\Omega,u,\omega))\right] + kD_\Omega\left[\sigma(J(\Omega,u,\omega))\right] = \sum_{i=1}^{n} D_\Omega\left[\mu_{J\_i}\right] + k\sum_{i=1}^{n} D_\Omega\left[\sigma_{J\_i}^2\right]$$

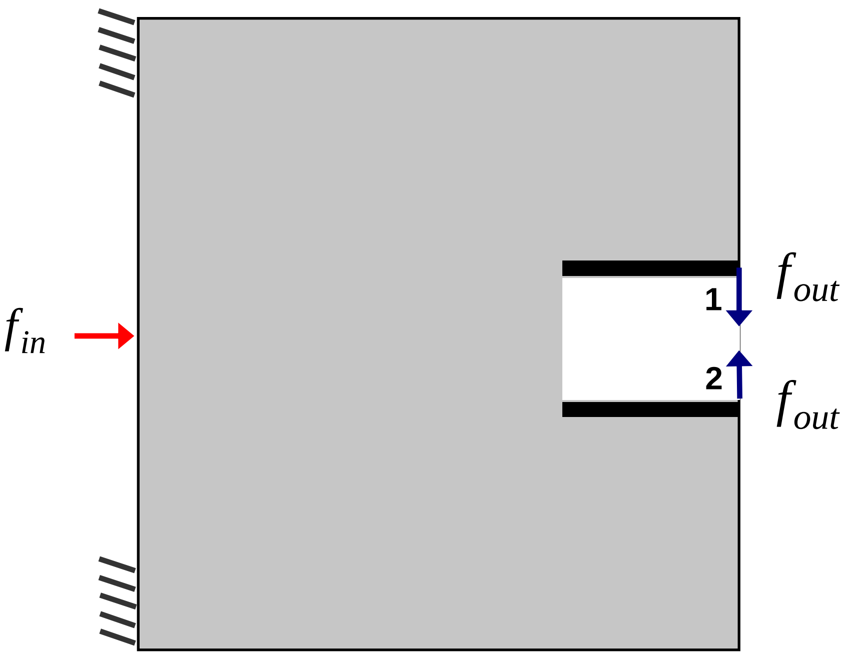
$$= \int_\Gamma \left\{ \frac{\partial J}{\partial u_{1i}} E_{ijkl}\varepsilon_{ij}(u_1)\varepsilon_{kl}(u_1) + \left(\frac{\partial J}{\partial u_{1o}} + \frac{\partial J}{\partial u_{2i}}\right) E_{ijkl}\varepsilon_{ij}(u_1)\varepsilon_{kl}(u_2) + \frac{\partial J}{\partial u_{2o}} E_{ijkl}\varepsilon_{ij}(u_2)\varepsilon_{kl}(u_2) \right\} V_n\, ds$$

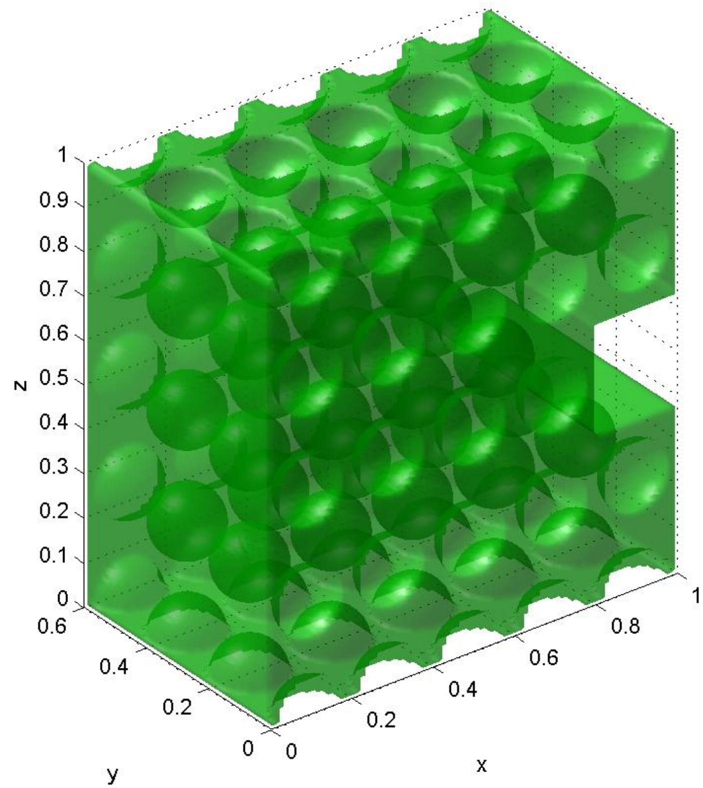
$J_\Omega^* = \int_\Gamma \beta(u) V_n\, ds$ → **Steepest Descent** → $V_n \triangleq -\beta(u)$ → $\varphi_t + V_n |\nabla\varphi| = 0$

Flowchart of the Level-Set-Based RSTO Algorithm



Example. Designing A Micro Gripper under A Random Material Field





RTO v.s. DTO

	Parameters	Volume Ratio	Robust Design	Deterministic Design
Material Field 1	$E = 1$	0.090	**-0.065**	-0.07
	$\mu_E = 1$ $\sigma_E = 0.3$ $d = 0.5$	0.098	**-0.059**	-0.055

Conclusion

- A LSM-based method to implement robust topology optimization;
- A gauss-type quadrature formula to calculate the mean and variance of the performance function;
- An adjoint method to derive the shape sensitivity of the mean and variance of the performance function;
- the results from RTO may be quite different from that of the deterministic topology optimization, **but…**
- Uncertainty is not the only factor that affect the topology of the final design. The interaction between the boundary condition and the uncertainties determines the topology of the final design to a large extent (keeping other conditions fixed).

Thanks!