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A Game Theory Based Composite Subspace Uncertainty Multidisciplinary Design Optimization Procedure

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Abstract
To enhance airplane design quality, the theory and application of Uncertainty Multidisciplinary Design Optimization (UMDO) is systematically studied in this paper. Firstly, the UMDO theory is briefly introduced. Secondly, the UMDO procedure is comprehensively studied. Based on the Concurrent SubSpace Optimization (CSSO) procedure, a Game theory Based Composite SubSpace Uncertainty Optimization (GBCSSUO) procedure is proposed. Multi-protocols are utilized synthetically to organize discipline relationship of subspace optimization of CSSO, so that the discipline organization can be more rational and flexible. An example is used to test the efficacy of the proposed method. Thirdly, the application of UMDO in airplane system design is studied. GBCSSUO is adopted to integrate and resolve the UMDO problem of a passenger airplane. The optimization result confirms the feasibility and validity of the proposed method, and offers a robust and reliable optimum system design scheme of the passenger airplane. Finally, the research work in this paper is summarized.
Keywords: Uncertainty Multidisciplinary Design and Optimization; UMDO Procedure; Concurrent Subspace Uncertainty Optimization; Game Theory; Airplane System Design

1. Introduction

For complex engineering systems, Multidisciplinary Design Optimization (MDO) is an effective method to solve the highly coupled design problems, as it can make full use of the inner multidisciplinary coupling relationship of systems and achieve optimum designs. In the realistic world, uncertainties exist objectively and greatly influence the design and manufacture. If uncertainties are carefully taken into consideration in the design phase, the reliability and robustness of the product would be efficiently enhanced. So in the traditional MDO, uncertainties should be reasonably considered and treated, and Uncertainty Multidisciplinary Design Optimization (UMDO) method should be adopted to solve the complex engineering design problems [1-3]. The difficulties of UMDO mainly include calculation burden and optimization efficiency. Lots of research work has been done to solve these problems, such as efficiency improvement of UMDO procedure and optimization solving method, accuracy improvement of approximation models and uncertainty analysis, etc. In this paper, the research work is focused on UMDO procedure.

2. UMDO Theory

UMDO is referred to the method which is used to solve the uncertainty design optimization problem of complex system by fully considering the coupling relationship and uncertainty propagation between disciplines. By utilization of UMDO procedure which integrates disciplinary models, uncertainty analysis methods, and optimization methods effectively, and by use of design of experiment (DOE) and approximation modeling methods, the calculation burden and organization complexity can be greatly reduced. For an uncertainty multidisciplinary design optimization problem, the general flowchart of solving procedure is depicted in Figure 1. From the diagram we can see that the solving process is mainly consisted of the following two parts.

(1) Uncertainty system modeling

Uncertainty system modeling includes system modeling and uncertainty modeling. System modeling refers to the mathematical modeling procedure of system and disciplines, and mathematical description of optimization problem, including the design variables, optimization objectives, robust performance objectives, constraints and reliability requirements, etc. Uncertainty modeling refers to the classification and quantification of uncertainties with uncertainty mathematical theory and methods [4, 5]. To simplify uncertainty problem and reduce calculation burden, it is generally necessary to utilize sensitivity analysis to screen out the factors which have significant importance on system design.

(2) UMDO procedure [6]

UMDO procedure refers to the executive sequence of system analysis, system decomposition, DOE, approximation modeling, design space searching algorithm, uncertainty analysis etc [7, 8]. It is the organization method of UMDO realization in computing environment. The reasonableness and effectiveness of the UMDO procedure directly influence the performance of UMDO. It is an important method to reduce calculation burden and improve optimization efficiency. So it is the research focus in this paper.

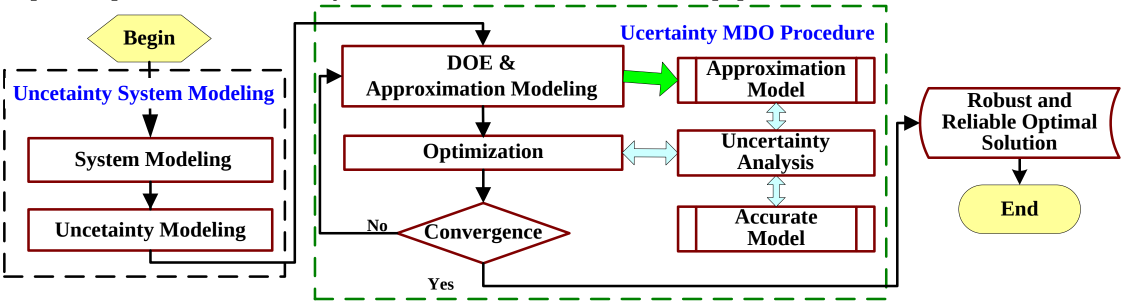


Figure 1: Flow Chart of Uncertainty Optimization

3. GBCSSUO Method

The UMDO procedure greatly influences the optimization efficiency of UMDO, especially in the non-hierarchical complex system design and optimization problem. To develop the UMDO procedure, the traditional deterministic MDO procedures can be used for reference [9, 10]. In this paper, the deterministic MDO procedure CSSO is referenced. Since CSSO was proposed by Sobieski in 1988, it has been widely studied and improved. In 1996, Sobieski and Batill proposed Response Surface based CSSO (CSSO-RS), which decouples disciplines by means of approximation models and improves CSSO's application agility [11]. CSSO has been successfully applied in many complicated engineering design and optimization problems. It realizes the decoupling of complicated disciplinary relationship and enables each discipline designer to design and optimize with their own unique and advanced tools independently. However, it has ignored the following two issues in the realistic design process:

- Different disciplines may have quite different influence on system performance. In other words, each discipline has different degree of importance on system design and optimization.
- The dependent or coupled degree of different disciplines may be quite different.

But in CSSO procedure, all the subspaces are equally treated in the subsystem level optimization. All the disciplines are decoupled from each other with the same method and optimized concurrently with the same importance. The ignorance of inherent distinction between disciplines may greatly influence the feasibility and validity of subsystems organization, and further exert influence on the optimization efficiency. For example, if one discipline has outstanding importance on the system design and others' are very weak, it is much more feasible to have the others' design work done following the "chief" one's design than have all the disciplines designed at the same time and then make the designs from each discipline consistent through repetitious iterations of system level coordination. To address this problem, we resort to game theory to ameliorate the organization form of subsystems.

3.1 Game Theory [12-13]

Game theory has typically been used in economics and business. In recent research, it began to be applied in system design. In a system perspective, a game consists of multiple decision-makers who each control a specified subset of system variables and who each seek to minimize their own cost functions subject to their individual constraints. Game theory is the study of the strategic interaction (decision strategies) of players in such games. From the definition above it is clear that a design process with multiple disciplinary teams is very similar with a "game", so researchers abstract design processes as games and disciplinary design teams with their associated analysis/synthesis tools as players. By exploring multi-player strategic interaction models, researchers try to improve the design efficiency and enlarge design profits in complex multidisciplinary design.

There are three fundamental game protocols applicable to design, including cooperative, noncooperative, and sequential (Stackelberg leader/follower) constructs.

- Cooperative: This model means complete cooperation occurs when each designer is aware of all the others and the decisions made by each. For system design, it means all the designers share their information and the transfer of information is seamlessly. Cooperative construct can be further divided into two types. One is Full Cooperation, and the other one is Approximate Cooperation. The former one means full information communication between disciplines, corresponding to all-in-one optimization procedure in MDO. The latter one means that discipline designer carries out work independently, and obtain information needed from other disciplines by means of global sensitive equation (GSE) or other approximation models. It resembles CSSO. The cooperative design solution is also called Pareto solution, which means all the disciplinary designers can't improve their own objectives simultaneously.

- Noncooperative: In this model, design teams usually act in their own interest. They may not get the necessary information they need to make decisions, so each design team has to make assumptions, many times worst case, about the information they need from other teams because of any kind of information isolation. This model is also known as Nash formulation, and the design solution is called the Nash solution. In real engineering design situations, different discipline teams may have their own optimization objectives. But in a systematic perspective, they also act as a whole collectivity and have a common goal to achieve, which is the performance of the whole system. So it is quite different from the noncooperative model, in which different teams only consider about their own interest and make decision to maximize their own profits rather than the interest of the whole group. So in this paper, we don't discuss this model.
- Sequential (Stackelberg leader/follower): In this model, the dominant leader team firstly makes decision, and then passes it to the next team. The leader needs make some assumptions about the behavior of its follower, and the follower makes its decision rationally based on the design result of its preceding team. The assumptions of the follower's behavior are made based on the rational reaction set (RRS), which also is the only link between the leader and the follower. RRS can be considered as a mapping from the leader's decision to the follower's reaction set. The math expression is

$$RSS(x_{leader}) = \{x^*_{follower} \in \mathbf{X}_{follower} \mid f_{follower}(x_{leader}, x^*_{follower}) = \min_{x_{follower} \in \mathbf{X}_{follower}} f_{follower}(x_{leader}, x_{follower})\} \qquad (1)$$

where $f_{follower}$ is the optimization objective of the follower discipline, x is the design decision and $\mathbf{X}$ is the design space. The optimization model of the leader can be written as

$$\begin{aligned} \min \quad & f_{leader}(x_{leader}, x_{follower}) \\ s.t. \quad & x_{follower} \in RSS(x_{leader}) \end{aligned} \qquad (2)$$

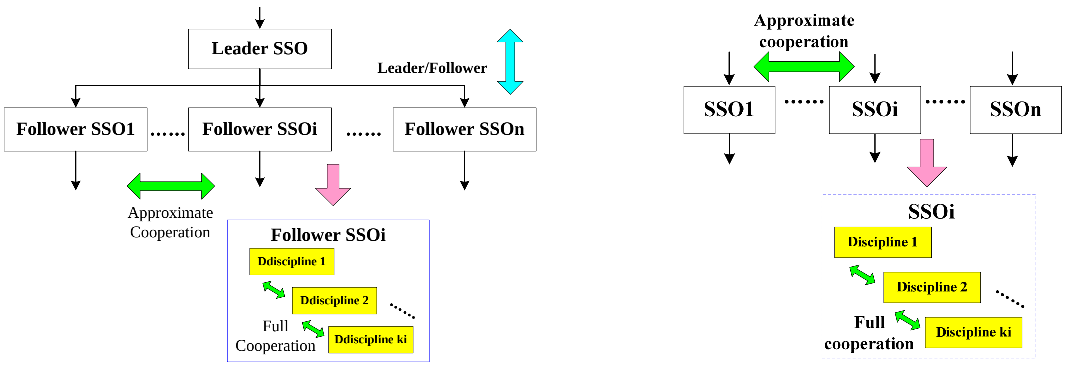
The RRS can be constructed by DOE and approximation approach.

Considering system design practice, only cooperative and sequential protocols are studied in this paper.

3.2 GBCSSO Procedure

To utilize game theory protocols to organize disciplines in subspace optimization of CSSO, the characteristics of each protocol should firstly be analyzed. Full cooperation can consider all the disciplines at the same time, but the calculation burden is very great in complex system design problem. So this type is only feasible for the optimization with very few disciplines and close coupled relationship. Approximate cooperation can improve discipline independency and reduce calculation time by concurrent discipline design, but it augments the coordination complexity as well, which may greatly decrease optimization efficiency. Sequential type costs much more time in a single optimization circle than the concurrent optimization, but it can maintain the design independency of the leader and improve the design consistency between the leader and the follower. So it can mitigate the coordination complexity and may reduce the whole optimization cycle number, so as to decrease the calculation burden. To sum up, each protocol has its merits and demerits. We should synthesize multi-protocols to make full use of each type's advantages and organize discipline relationship effectively according to specific design situation. This method we propose is called game theory multi-protocol based discipline organization method.

According to above analysis of protocols, we propose two types of multi-protocol integration forms. It is illustrated in Figure 2.



(a) (b)

Figure 2: Composite subspace organization relationship diagram

In form (a), there is one leader subspace and several followers. The relation among follower subspaces is approximation cooperation. Each follower subspace may be consisted of several disciplines. The relation among these disciplines is full cooperation, which means these disciplines are very close coupled. In subspace optimization step of CSSO, the leader subspace firstly runs optimization. Then the followers run optimization concurrently. In form (b), there is no leader subspace. All the disciplines are grouped into several subspaces. The relation among subspaces is approximation cooperation, and the relation among disciplines within a subspace is full cooperation. This form is the same with the subspace organization of traditional CSSO. These two types can be applied to different optimization problem according to specific situation.

With above two composite protocol organization forms, the discipline organization in subspace optimization can be more rational and as a result the optimization efficiency can be enhanced. This improved procedure we propose is named Game theory Based Composite SubSpace Optimization (GBCSSO) procedure.

3.3 GBCSSUO Procedure

Based on the deterministic MDO procedure GBCSSO proposed above, further integrating uncertainty design and optimization method, we put forward a UMDO procedure called Game theory Based Composite SubSpace Uncertainty Optimization (GBCSSUO). The flowchart of GBCSSUO is shown in Figure 3.

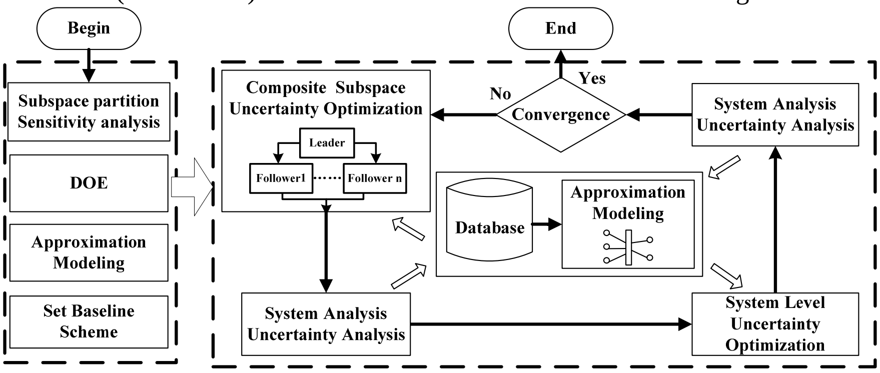


Figure 3: Flow chart of GBCSSUO

The detail of each block in the diagram is described in the following.

Step 1: Partition the complex system into several subspaces. Analyze each subspace's distinction and coupled relationship with each other. If there is one subspace which dominates the others, this one is selected as the leader, and form (a) is chosen to organize the subspace optimization. If there is no subspace which is worthy of the leader position, then the form (b) is chosen. Sensitivity analysis method can be used to determine the influence of design variables of each subspace on the system design, so as to provide reference for selection of organization form. The sensitivity analysis is also applied to study the importance of uncertainty parameters. The significant ones are screened out so as to decrease the uncertainty optimization burden.

Step 2: Construct approximation models with DOE techniques. The approximation models include three classes. The first class is the model used as replacement of high fidelity system models, so as to reduce calculation complexity during optimization and uncertainty analysis. The second class is the model used as decoupling method in subspaces with approximation cooperation relationship. The third class is the RSS used in leader-follower relationship.

Step 3: Set the baseline for optimization. The selection of baseline is very important to optimization efficiency, especially for UMDO. In some cases, we can firstly run deterministic MDO procedure to get a primary better design, and then run the UMDO procedure to get the optimum under uncertainties.

Step 4: System optimization. The math model is

$$\begin{cases} Find & \mathbf{X} \\ Min & f_{UO} = k_1 \tilde{\mu}_f\left(\mathbf{X},\mathbf{p}\right) + \sum_{l=1}^{r} k_{l+1} \tilde{\sigma}_{\mathbf{Y}_{robust}(l)}(\mathbf{X},\mathbf{p}) \\ S.t. & P\{\mathbf{g}\left(\mathbf{X},\mathbf{p}\right) \le 0\} \ge R_g \\ & P\{\left|\mathbf{h}\left(\mathbf{X},\mathbf{p}\right)\right| \le \varepsilon\} \ge R_h \\ & \mathbf{X}^L + \Delta\mathbf{X} \le \mu_{\mathbf{X}} \le \mathbf{X}^U - \Delta\mathbf{X} \end{cases} \tag{3}$$

f_{UO} is the optimization objective, which is the weighted sum of several sub-objectives including minimizing the expected mean value of the objective function and the standard deviation of system state variables $\mathbf{Y}_{robust}$ which have robust design requirement. $\mathbf{p}$ is the system parameter vector. $\Delta\mathbf{X}$ is design variation range. R_g and R_h are reliability requirements for inequality and equality constraints respectively. ε is a small quantity which limits the deviation range of the equality constraint. "~" means the value is calculated by approximation model.

Step 5: Subsystem optimization. In organization form (a), the leader subspace optimization model is

$$\begin{cases} Find & \mathbf{X}_{leader} \\ Min & f_{UO_{leader}} = k_1\mu_{f_{leader}}\left(\mathbf{X}_{leader}, \bar{\mathbf{X}}_i, \mathbf{Y}_{leader}, \tilde{\mathbf{Y}}_i, \mathbf{p}_{leader}\right) \\ & \qquad + \sum_{l=1}^{r_{leader}} k_{l+1}\sigma_{\mathbf{Y}_{robust-leader}(l)}\left(\mathbf{X}_{leader}, \bar{\mathbf{X}}_i, \mathbf{Y}_{leader}, \tilde{\mathbf{Y}}_i, \mathbf{p}_{leader}\right) \\ & i = 1, 2, \cdots n \\ S.t. & P\{\mathbf{g}_{leader}\left(\mathbf{X}_{leader}, \bar{\mathbf{X}}_i, \mathbf{Y}_{leader}, \tilde{\mathbf{Y}}_i, \mathbf{p}_{leader}\right) \le 0\} \ge R_g \\ & P\{\left|\mathbf{h}_{leader}\left(\mathbf{X}_{leader}, \bar{\mathbf{X}}_i, \mathbf{Y}_{leader}, \tilde{\mathbf{Y}}_i, \mathbf{p}_{leader}\right)\right| \le \varepsilon\} \ge R_h \\ & \mathbf{X}_{leader}{}^L + \Delta\mathbf{X}_{leader} \le \mu_{\mathbf{X}_{leader}} \le \mathbf{X}_{leader}{}^U - \Delta\mathbf{X}_{leader} \end{cases} \tag{4}$$

$\bar{\mathbf{X}}_i$ is the design vector of the i th subspace whose value is fixed according to system design. $\tilde{\mathbf{Y}}_i$ is the state vector of the i th subspace whose value is calculated though RSS of this subspace. n represents the number of the follower subspaces.

The optimization model of the i th follower subspace is

$$\begin{cases} Find & \mathbf{X}_i \\ Min & f_{UO_i} = k_1\mu_{f_i}\left(\mathbf{X}_i, \bar{\mathbf{X}}_j, \mathbf{X}_{leader}, \mathbf{Y}_i, \tilde{\mathbf{Y}}_j, \mathbf{Y}_{leader}, \mathbf{p}_i\right) \\ & \qquad + \sum_{l=1}^{r_i} k_{l+1}\sigma_{\mathbf{Y}_{robust-i}(l)}\left(\mathbf{X}_i, \bar{\mathbf{X}}_j, \mathbf{X}_{leader}, \mathbf{Y}_i, \tilde{\mathbf{Y}}_j, \mathbf{Y}_{leader}, \mathbf{p}_i\right) \\ & i = 1, 2, \cdots n; \quad j = 1, 2, \cdots n, j \ne i \\ S.t. & P\{\mathbf{g}_i\left(\mathbf{X}_i, \bar{\mathbf{X}}_j, \mathbf{X}_{leader}, \mathbf{Y}_i, \tilde{\mathbf{Y}}_j, \mathbf{Y}_{leader}, \mathbf{p}_i\right) \le 0\} \ge R_g \\ & P\{\left|\mathbf{h}_i\left(\mathbf{X}_i, \bar{\mathbf{X}}_j, \mathbf{X}_{leader}, \mathbf{Y}_i, \tilde{\mathbf{Y}}_j, \mathbf{Y}_{leader}, \mathbf{p}_i\right)\right| \le \varepsilon\} \ge R_h \\ & \mathbf{X}_i{}^L + \Delta\mathbf{X}_i \le \mu_{\mathbf{X}_i} \le \mathbf{X}_i{}^U - \Delta\mathbf{X}_i \end{cases} \tag{5}$$

During optimization process, the value of $\mathbf{X}_{leader}$ and $\mathbf{Y}_{leader}$ is fixed. The state vector $\tilde{\mathbf{Y}}_j$ of the j th subspace is calculated by means of the approximation models. The subscript $\mathbf{Y}_{robust-i}$ means the system state variables within the i th subspace which have robust design requirement.

In organization form (b), the subspace optimization model is the same as the CSSO.

All through the system and subsystem optimization, there is uncertainty analysis to get the robustness and reliability information of the design. There are several uncertainty analysis methods, including Monte Carlo analysis, Taylor series approximation, First Order Reliability Method (FORM), Second Order Reliability Method (SORM), Mean Value First Order Method (MVFO) etc.

Step 6: System analysis and uncertainty analysis. This step is to coordinate the design of subspaces, and analysis the design with high fidelity discipline models which are too expensive to run in the optimization step. The function of uncertainty analysis is the same with that in optimization. But in this step, the accuracy of the analysis result is paid more attention, so simulation method is usually adopted which has advantage in accuracy but too time-consuming to use in optimization.

4. Test and Application

In this section, test and application of the proposed method GBCSSUO are discussed. Firstly, a reducer design problem is used to test GBCSSUO. Secondly, GBCSSUO is applied in an airplane system design problem, so as to explore the application of UMDO in aeronautical engineering.

4.1 Example 1: Speed Reducer Design

Before we apply the proposed UMDO method to airplane design, we firstly use a mathematical problem to validate this method. The problem is a speed reducer design which is one of the standard test problems proposed by NASA MDO Branch to evaluate MDO procedure [14]. The math model is

$$
\begin{aligned}
\textit{Minimize}:\ & f(x)=0.7854x_1x_2^2(3.3333x_3^2+14.9334x_3-43.0934)-1.5079x_1(x_6^2+x_7^2)\\
& +7.477(x_6^3+x_7^3)+0.7854(x_4x_6^2+x_5x_7^2)\\
\textit{Subject to}:\ & g_1:27.0/(x_1x_2^2x_3)-1\le 0,\ \ g_2:397.5/(x_1x_2^2x_3^2)-1\le 0\\
& g_3:1.93x_4^3/(x_2x_3x_6^4)-1\le 0,\ \ g_4:1.93x_5^3/(x_2x_3x_7^4)-1\le 0\\
& g_5:A_1/B_1-1100\le 0,\ \ g_6:A_2/B_2-850\le 0\\
& g_7:x_2x_3-40.0\le 0,\ \ g_8:5.0\le x_1/x_2\le 12.0\\
& g_9:(1.5x_6+1.9)/x_4-1\le 0,\ \ g_{10}:(1.1x_7+1.9)/x_5-1\le 0\\
& A_1=\left[\left(745.0x_4/x_2x_3\right)^2+16.9\times 10^6\right]^{0.5},B_1=0.1x_6^3\\
& A_2=\left[\left(745.0x_5/x_2x_3\right)^2+157.5\times 10^6\right]^{0.5},B_2=0.1x_7^3\\
& 2.6\le x_1\le 3.6, 0.7\le x_2\le 0.8, 17\le x_3\le 28, 7.3\le x_4\le 8.3\\
& 7.3\le x_5\le 8.3, 2.9\le x_6\le 3.9, 5.0\le x_7\le 5.5
\end{aligned}
\tag{6}
$$

The description of each variable is illustrated in Table 1. We suppose the manufacturing mismatching tolerance is normal distributed and the standard tolerance degree is seven, so the variance of the uncertainty design variable is calculated according to its size range. The uncertainty analysis result is also listed in Table 1.

Table 1: Uncertainty design variables of example 1

Variable Name	Symbol	Distribution	Variance
gear face width /cm	x1	Normal	21um
teeth module /cm	x2	Normal	1um
number of teeth of pinion	x3	/	/
Distance between bearings 1/cm	x4	Normal	30um
Distance between bearings 2/cm	x5	Normal	30um
diameter of shaft 1/cm	x6	Normal	21um
diameter of shaft 2/cm	x7	Normal	30um

Based on the research results of subspace partition in this design problem by Yong Zhao [6], we define the subspaces partition as follows.

Subspace 1（SSO1）: $X_1=\{x_1,x_2,x_3\}$

Subspace 2（SSO2）: $X_2=\{x_4,x_6\}$

Subspace 3（SSO3）: $X_3=\{x_5,x_7\}$

Sensitivity analysis of the variables is performed, and the result is shown in Figure 4. It's obvious from the chart that x_1 and x_3 have significant importance on the objective. In other words, the variables of subspace one have outstanding effect on the objective, so we use the form (a) to organize subspaces. Subspace one is chosen as the leader and the other two subspaces are followers. The subspace relationship diagram is shown in Figure 5.

The optimization problem is firstly solved with GBCSSO and CSSO respectively without consideration for uncertainties. On one hand we can test the utility of game theory compared to the traditional CSSO. On the other hand, we can provide a relative better baseline for further uncertainty optimization so as to reduce computing burden. The results are listed in Table 2, and the optimization iteration convergence history is plotted in Figure 6.

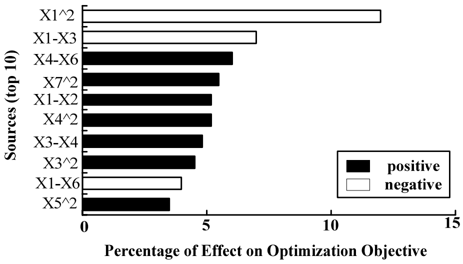


Figure 4: %Effect of design variables on objective in example 1

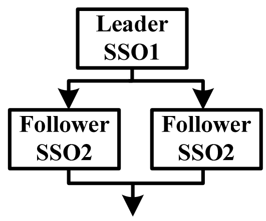
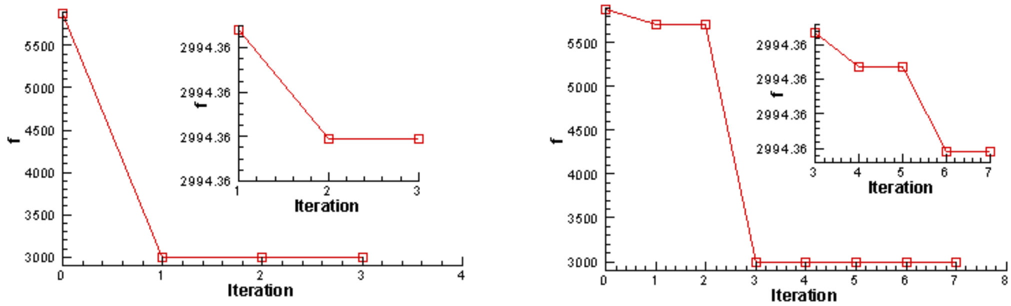


Figure 5: Subspace relationship diagram in example 1

Table 2: Comparison of optimization results of GBCSSO and CSSO in example 1

	GBCSSO	CSSO
Start point	3.6, 0.7, 28, 8.3, 8.3, 3.9, 5.5	
Optimum	3.5, 0.7, 17, 7.3, 7.715325, 3.350218, 5.2866545	3.51, 0.7, 17, 7.3, 7.715324, 3.350215, 5.286655
Objective	2994.35660	2994.35587
Iteration Number	3	7
Time /s	85	250



(a) GBCSSO optimization iteration history (b) CSSO optimization iteration history

Figure 6: Comparison of optimization convergence history of GBCSSO and CSSO in example 1

From Table 2 we can see that optimization results of the two procedures are almost the same. But in perspective of convergence iteration number and optimization time, GBCSSO has obvious advantage over CSSO. This proves validity of GBCSSO.

We further consider uncertainties of design variables and set reliability requirement of each constraint to be 95%. The optimization objective is minimizing the expected value of the objective function. GBCSSUO is adopted to solve the UMDO problem. The optimization result is compared with that of GBCSSO and listed in Table 3.

Table 3: Comparison of optimization results of GBCSSUO and GBCSSO in example 1

		GBCSSUO	GBCSSO
Start Point		3.6, 0.7, 28, 8.3, 8.3, 3.9, 5.5	
Optimum		3.534808628, 0.7, 17, 7.3, 7.8951811, 3.3837628, 5.3396063	3.5, 0.7, 17, 7.3, 7.7153199, 3.3502147, 5.2866545
Objective		3054.6174	2994.3566
Constraints	Pr（g1≤ 0）	1	1
	Pr（g2≤ 0）	1	1
	Pr（g3≤ 0）	1	1
	Pr（g4≤ 0）	1	1
	Pr（g5≤ 0）	1	0.498
	Pr（g6≤ 0）	1	0.5
	Pr（g7≤ 0）	1	1
	Pr（g8≤ 0）	1	0.494
	Pr（g9≤ 0）	1	1
	Pr（g10≤ 0）	1	1
	Pr（g11≤ 0）	1	0.492
Time /s		2580	85

From Table 3 it can be seen that optimization result of GBCSSUO is not as good as GBCSSO. But reliabilities of all constraints get to one satisfying the 95% requirement, while four constraint reliabilities are lower than 0.5 in GBCSSO failing to meet the reliability requirement. In the table the reliability is equal to one means that the constraint can be satisfied with the probability of one. GBCSSUO sacrifices expectation of optimization objective to guarantee reliability satisfaction of every constraint, while deterministic MDO method optimized the design with the danger of violating constraints under uncertainties. Meanwhile, the time cost of GBCSSUO is obviously much longer than GBCSSO, which demonstrates the calculation complexity of UMDO. This example clearly validates the feasibility of GBCSSUO in efficiently achieving optimum design and meanwhile improving design's reliability.

4.2 Example 2: Design of a Passenger Airplane

To apply the proposed UMDO procedure GBCSSUO in aeronautic engineering, the conceptual system design problem of a passenger airplane is studied. There are three disciplines considered, including propulsion, aerodynamics and weight. For simplification of conceptual system design, the engineering estimation analytical models are utilized [15]. The coupled disciplinary relationship is described in Figure 7.

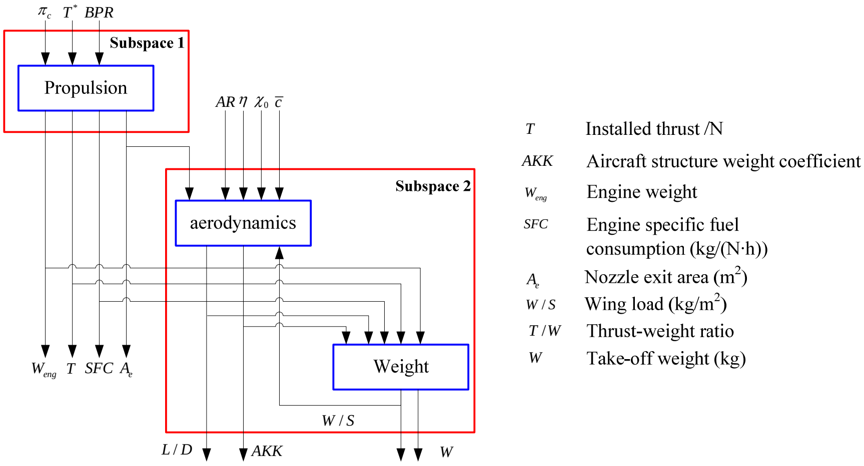


Figure 7: Coupled disciplinary relationship of passenger airplane system design model

According to the data flow, it is obvious that the weight discipline has no design variables. It only calculates airplane take-off weight and wing load based on the output from the other two disciplines. So we can consider it as a pure analysis module. Wing load of this discipline is the input of aerodynamics, so these two are coupled. The propulsion has one variable output nozzle exit area which is the input of aerodynamics, and has no input which is also the output of the other two disciplines. So it is unilaterally independent of the other two. Naturally we divide them into two subspaces. One includes propulsion, and the other one includes aerodynamics and weight. It is also quite obvious that the relationship between these two subspaces is sequential. So in GBCSSUO the two subspaces are organized in leader/follower type, rather than the concurrent type organized by the CSSO.

The optimization objective is to minimize the expectation of airplane take-off weight under uncertainties. The design constraints are to limit the wing load and thrust-weight ratio within required range with required reliability. The design variables include airfoil aspect ratio, leading edge sweeping angle, airfoil relative thickness, airfoil taper ratio, bypass ratio, engine compression ratio, and engine turbine temperature. The uncertainty characteristics of these variables are analyzed in Table 4. System parameters are considered as constants.

For discussion convenience, all the data are normalized. For each design variable and constraint, the upper and lower limits are normalized as one and zero respectively. For objective function, its value in optimization start point is chosen as the baseline and normalized as one. Other value is compared with this baseline and the ratio between them is taken as the normalization result.

Table 4: Uncertainty design variables of example 2

Name	Design range	Distribution	Coefficient. of Variance
airfoil aspect ratio AR	[0,1]	Normal	0.006
leading edge sweeping angle χ_0	[0,1]	Normal	0.0002
airfoil relative thickness $\bar{c}$	[0,1]	Normal	0.01
airfoil taper ratio η	[0,1]	Normal	0.006
bypass ratio BPR	[0,1]	Normal	0.01
engine compression ratio π_c	[0,1]	Normal	0.01
engine turbine temperature T_3	[0,1]	Normal	0.01

The uncertainty optimization mathematical model is

$$
\begin{aligned}
&Find: && \mathbf{X} = [AR, \chi_0, \bar{c}, \eta, BPR, \pi_c, T^*]^T \\
&Minimize: && W_{TO} = E[f(\mathbf{X})] \\
&Subject\ to: && P_r\{T/W \in [0,1]\} \geq 0.95 \\
& && P_r\{W/S \in [0,1]\} \geq 0.95 \\
& && \mathbf{X} \pm \Delta\mathbf{X} \in \mathbf{D}
\end{aligned} \tag{7}
$$

To reduce calculation burden of UMDO, deterministic MDO procedure is firstly run to achieve a preliminary better design solution as the start for UMDO. The optimization problem is solved with GBCSSO and CSSO respectively so as to compare the efficiency of game theory in subspace organization. The optimization results are listed in Table 5, and the optimization iteration convergence history is plotted in Figure 8. From Table 5, it is clear that both MDO procedures have achieved good optimization results. GBCSSO is slightly inferior to CSSO in objective minimization efficiency and the margin is 0.6%. But both the convergence cycle number and optimization time of GBCSSO are much less than those of CSSO. In other words, GBCSSO achieves almost the same optimization result with much less calculation cost. So in a synthetically view, GBCSSO is more efficient than CSSO. Based on the result of GBCSSO as the baseline, we continue to run GBCSSUO. The result is also listed in Table 5. From the results comparison of GBCSSUO and GBCSSO, it’s clearly that the reliability of the design scheme to meet constraint is greatly improved by considering uncertainties in the optimization.

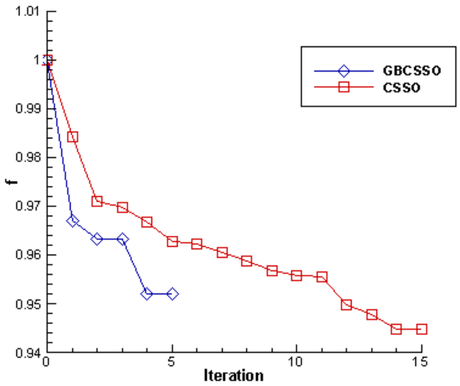


Figure 8: Comparison of optimization convergence history of GBCSSO and CSSO in example 2

Table 5: Comparison of optimization results of CSSO, GBCSSO and GBCSSUO in example 2

	Name	Start Point	CSSO	GBCSSO		GBCSSUO	
Design Variables	AR	0.33	0.21	0.22		0.22	
	$\bar{c}$	0.33	0.1	0.28		0.31	
	η	0.667	0.0	0.43		0.45	
	χ_0	1.0	1.0	0.98		0.98	
	BPR	0.167	0.024	0.305		0.328	
	π_c	0.50	0.155	0.326		0.326	
	T^*	0.33	0.92	0.67		0.78	
Constraints	T/W	0.38	0.0	0.02	Pr=0.65	0.06	Pr=1
	W/S	0.80	0.13	0.15	Pr=1	0.17	Pr=1
Objective	W /kg	1	0.945	0.951		0.956	
Others	Convergence cycle number	/	16	6		8	
	Optimization time (s)	/	4531	8390		79632	

5. Conclusions

As various kinds of uncertainties exist in the world objectively, it is very important to take uncertainty into consideration from the beginning of system design and optimize the system performance, meanwhile to improve robustness and reliability. To improve UMDO efficiency, UMDO procedure is studied in this paper.

GBCSSO procedure is based on the deterministic MDO procedure CSSO, and adopts game theory to organize the relationships of subspaces in subsystem level optimization. Considering the industry reality, only cooperation protocol and sequential protocol are synthetically utilized. Based on the protocol characteristics, the applicability of each protocol is analyzed, and two types of multi-protocol integration forms are proposed. One is consisted of one leader and several follower subspaces, and the other one is consisted of several equal subspaces. In the first form, the subspace with most significant importance on design is optimized firstly, which primarily determines the scheme and design direction, and then the followers continue to execute optimization based on the results passed down from the leader. This form can make full use of the sequential design advantage in keeping consistency in the design decision of the key variables, which can effectively reduce the calculation cost caused by disciplinary cooperation in independently concurrent design. In the second form, each subspace has nearly equal importance in design, so they can be treated equally and run optimization concurrently, which can make full use of parallel computation and save design optimization time. Each form can be applied in different situation according to specific characteristics of the UMDO problem. To sum up, this procedure can flexibly meet different design requirements and make full use of the intrinsic connection features between subspaces, so as to improve optimization efficiency.

GBCSSUO is proposed based on the efficient procedure GBCSSO and integration of the uncertainty theory and analysis methods. GBCSSUO can efficiently organize the process of complex system uncertainty design and optimization, and mitigate the calculation difficulties, so as to improve the optimization efficiency of UMDO. An example of speed reducer design is taken to test GBCSSUO, and the results validate its feasibility and efficiency. The GBCSSUO is further applied in a system design problem of a passenger airplane, and the results provide a reliable design scheme, which has some reference value for application of UMDO in airplane design.

In this paper, the UMDO of airplane is only primarily explored, and the effectiveness and feasibleness is confirmed. In the future, to improve the application of UMDO in airplane system design, the uncertainty modeling of airplane system, design models and manufacture should be studied in detail based on the industry and experiment data, and high fidelity accurate disciplinary models should be adopted in the optimization procedure rather than the simple engineering analytical models used in this paper. As computing complexity is ever increasing, the approximation methods applicable to high dimension and high nonlinear models should be utilized. GBCSSUO can also be applied in airplane subsystem or components design optimization problem, so as to enhance airplane performance from bottom to top.

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