Effects of Electrostatic Instability on the Performance of MEMS Actuators

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ABSTRACT

This paper examines electrostatic instability in a 1DOF MEMS micro mirror and the effect this has on the device performance. As with many electrostatic actuators, this micro mirror exhibits electrostatic pull-in. Analyzing the fixed-point solutions of the system and creating a bifurcation diagram determine the pull-in angle and voltage to be 16.5 degrees and 132.71 volts, respectively. Once the mirror is in its pulled-in position, the holding voltage and hysteretic behavior can be predicted by evaluating the electrostatic and mechanical forces applied to the actuator. From this analysis, the holding-voltage is determined to be 124 volts.

Keywords

MEMS, electrostatic instability, bifurcation, hysterisis

1. INTRODUCTION

Micro mirrors have emerged as key components for optical microelectromechanical system (MEMS) applications, including switching and scanning operations. Electrostatic vertical comb drive actuators are attractive because they can be fabricated underneath the mirror, allowing for arrays with a high fill factor. Also, vertical comb drives are more easily controlled than parallel plate actuators, making them the better choice for analog scanning devices [1-6].

One limiting factor to most electrostatic actuators is the electrostatic pull-in instability that occurs when the electrostatic force overcomes the mechanical restoring force. When pull-in occurs, the device can no longer maintain an equilibrium position and will move to its fully actuated position, limiting the full scanning range available. Another phenomenon associated with this instability is that once the mirror has pulled-in, the voltage required to maintain that position is lower than the pull-in voltage. The mirror will not return from this position until the actuating voltage has been reduced below the holding-voltage. The result of this holding effect is hysterisis.

The device presented in this paper is a one-degree of freedom vertical comb drive fabricated by surface micromachining. The electrostatic performance of the device is investigated by determining the stability of the fixed point solutions as the applied voltage is varied. From this analysis, the bifurcation point is determined that corresponds to the electrostatic pull-in voltage and angle. In addition, by examining the static equilibrium of the equations of motion, the hold-down voltage and hysterisis behavior of the device can be estimated.

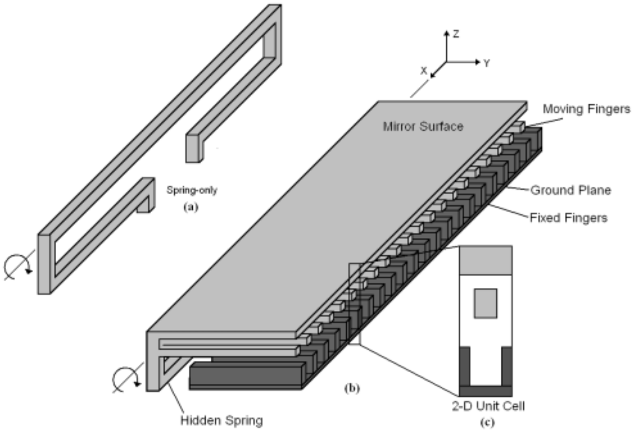

Figure 1. Schematic of 1DOF micro mirror device with hidden vertical comb fingers.

2. STABILITY ANALYSIS

For the electrostatic comb drive mirror presented here, there is a ground plane and a series of vertically offset comb fingers, all contained underneath a flat mirror surface. A voltage potential is applied across the fixed fingers and the moving fingers of the device creating an electrostatic force. This force causes the mirror to rotate about an axis supported by the hidden spring suspension, shown separately in Figure 1.

The equation of motion is

$$m\ddot{\theta} + b\dot{\theta} + k\theta = T_e(\theta) \tag{1}$$

where m is the mass of the plate, b is the damping term due to squeeze-film effect, k is the linear spring constant, and $T_e(\theta)$ is the electrostatic torque. The determination of squeeze-film damping coefficient is dependent upon the geometry of the surfaces between which the fluid is trapped. Because of the vertical comb fingers under the surface of the mirror, determining this coefficient analytically would be difficult. So for the purpose of this discussion, consider the squeeze-film damping term for a torsional plate developed by Pan, et al [7].

$$b = K_{rot} \frac{\mu L w^5}{g^3} \tag{2}$$

where

$$K_{rot} = \frac{48}{\pi^6 \left[\left(\frac{w}{L}\right)^2 + 4 \right]} \tag{3}$$

L is the length of the plate, w the width, g is the gap between the plates, and μ is the viscosity of the fluid.

The electrostatic torque T_e is given as

$$T_e = \frac{1}{2} N \frac{\partial C}{\partial \theta} V^2 \tag{4}$$

where V is the applied voltage, and C is the capacitance which can be determined from simulation to be a function of the rotation angle [6]. Because of the symmetry of the comb fingers, a unit cell is defined as shown in Figure 1, and N is the total number of unit cells. The results of the capacitance for one unit cell are plotted in Figure 2 along with a fourth order polynomial approximation of the data. The capacitance function is

$$\begin{aligned} C(\theta) &= P(1)\theta^4 + P(2)\theta^3 + P(3)\theta^2 \\ &+ P(4)\theta + P(5) \end{aligned} \tag{5}$$

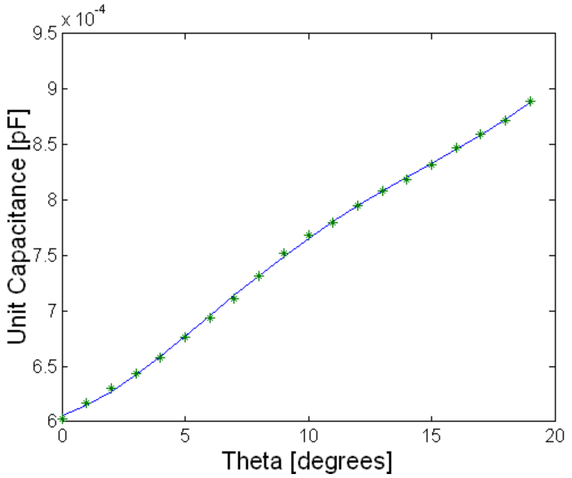

Figure 2. Capacitance calculation as a function of rotation angle, theta, calculated using 3D FEA (*) and a fourth order polynomial curve fit (line).

Table 1. List of parameters used for this analysis

Parameter	**Value**
ρ, density of polysilicon	2331 kg/m^3
k_1, linear spring constant	611.23 pN-um/rad
μ, viscosity of air	1.73e-5 N-s/m^2
L, length of mirror	20 μm
w, width of mirror	100 μm
g, gap between plates	11.25 μm
N, number of unit cells	27

where the coefficients of the polynomial are P = [0.0041 -0.0002 2.176 7.201 605.5] in pF. Table 1 lists the values of additional parameters for this system.

When the electrostatic torque is equal to the mechanical restoring torque, the system is in equilibrium. Once the electrostatic torque becomes greater than the mechanical restoring torque, electrostatic 'pull-in' occurs. The angle and voltage at which pull-in occurs can be determined by examining the stability of the fixed point solutions and creating a bifurcation diagram using the actuation voltage as the control parameter.

The state space model for the system is

$$\begin{aligned} x_1 &= \theta \\ x_2 &= \dot{\theta} \\ \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} x_2 \\ \frac{1}{m} T_e(x_1) - \frac{b}{m} x_2 - \frac{k_1}{m} x_1 \end{bmatrix} \end{aligned} \tag{6}$$

The fixed points occur at $x_2 = 0$ and

$$T_e(x_1) - k_1 x_1 = 0 \tag{7}$$

Solving this equation gives the fixed points as functions of the control parameter V. Taking the Taylor series expansion and retaining the first term only gives the Jacobian matrix.

$$Df(\bar{x}) = \begin{bmatrix} 0 & 1 \\ \frac{1}{m} \frac{\partial T_e(x_1)}{\partial x_1} - \frac{k_1}{m} & -\frac{b}{m} \end{bmatrix} \tag{8}$$

The stability is determined by evaluating (8) at the fixed points and determining the eigenvalues. The fixed point solution is stable when the real part of the eiqenvalues is less than zero. The

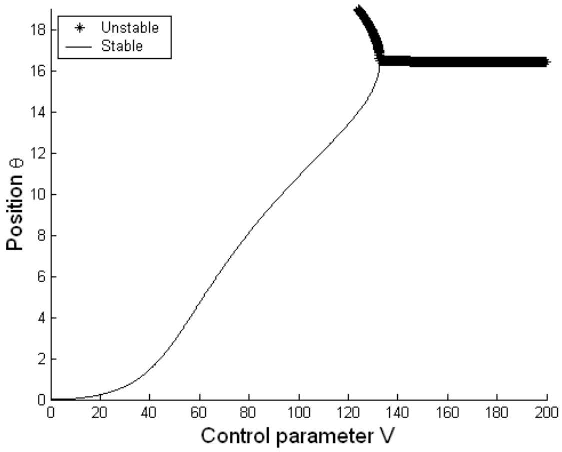

Figure 3. The bifurcation diagram for a MEMS torsion mirror with electrostatic vertical comb drive actuator shows that the bifurcation point occurs at 16.5 degrees and 132.71 V.

bifurcation diagram of this device is shown in Figure 3. The bifurcation occurs at 16.5 degrees and 132.71 V.

Hah, et al. derived an analytical relationship to determine the pull-in point. Assuming that the restoring springs are linearly deformed in the range of actuation, the pull-in can be described by

$$\left(\frac{\partial C}{\partial \theta}\right)_{\theta=\theta_{PI}} - \theta_{PI}\left(\frac{\partial^2 C}{\partial \theta^2}\right)_{\theta=\theta_{PI}} = 0 \qquad (9)$$

where θ_{PI} is the pull-in angle [5]. It is clear from this equation that the pull-in is independent of the spring stiffness. In turn, once the pull-in angle is determined, the pull-in voltage can be calculated by the following,

$$V_{PI} = \sqrt{\frac{2k_1\theta_{PI}}{\left(\frac{\partial C}{\partial \theta}\right)_{\theta=\theta_{PI}}}} \qquad (10)$$

Evaluating equations (9) and (10), the pull-in angle is found to be 16.5 degrees, and the corresponding pull-in voltage is 132.71 V. This agrees with the results from the bifurcation analysis above.

3. HYSTERISIS

The pull-in occurs at the bifurcation voltage, however once it is in the pulled-in position, the voltage required to hold it there is less than the pull-in voltage. This effect can be attributed to adhesion and surface forces [8]. The mirror will not release from the pulled-in position until the voltage is below this holding voltage, V_h and the result is hysterisis.

To gain a better understanding of this effect, the electrostatic torque in equation (4) can be calculated as a function of rotation angle and voltage. Figure 4 shows the resulting electrostatic torque from (4) as a function of rotation angle for different values of voltage ranging from 10 volts to 140 volts. The dashed, 's'-shaped curves on the plot indicate lines of constant voltage, and can be used to determine the position of the device at any voltage. Also on this plot is a straight line corresponding to the linear mechanical restoring torque. When the mechanical restoring torque is equal in value and slope to the electrostatic torque, pull-ocurs [5]. This is better shown in figure 5. For this device, pull-in occurs at a rotation angle of 16.5°, and a voltage of 132.71 volts, agreeing with the results from the instability analysis.

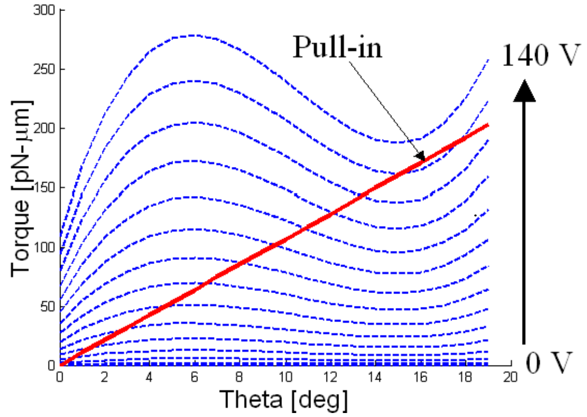

Figure 4. Electrostatic torque as a function of rotation angle, theta, and voltage determined from equation (4). The straight line shows the restoring torque from the linear springs.

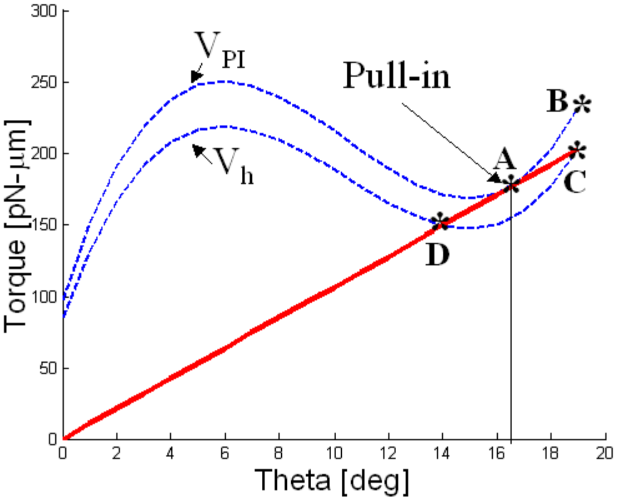

Figure 5. Plot describing the determination of the holding-voltage, V_h, and the resulting hysterisis behaviour.

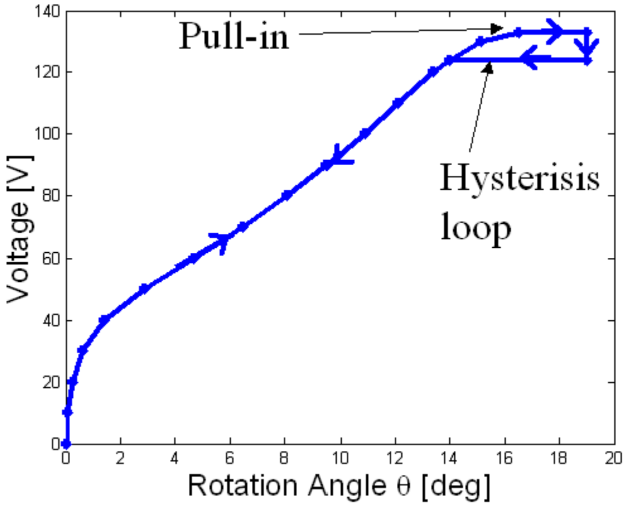

Figure 6. Plot of actuation voltage versus the rotation angle obtained from the figure 4 showing the pull-in point and hysterisis loop.

The information from figure 4 can be used to determine the equilibrium position of the mirror for a given voltage. Before the bifurcation point, the equilibrium position of the mirror can be found where the straight line crosses the constant voltage curves. At the bifurcation point, the mirror pulls-in to a position of 19 degrees. The holding voltage can be determined by finding the voltage value that corresponds to 19 degrees and the torque value at pull-in. This is illustrated in figure 5. At point ‘A’, the electrostatic torque becomes greater than the mechanical restoring torque from the spring, causing the mirror to pull-in and rotate to 19 degrees. The torque applied to the mirror is now indicated by point ‘B’. The voltage required to maintain this position is can be lowered to a holding-voltage, V_h. Point ‘C’ indicates the holding-voltage V_h is 124 V. Once the voltage drops below this value, the mirror will return to an equilibrium position at the reduced voltage value. This is what occurs at point ‘D’ and this accounts for the hysterisis effect.

Using the plots in figures 4 and 5, the relationship between the voltage and rotation angle is determined. During the forward actuaion, analog motion can be achieved for 0 to 16.5 degrees. Then, pull-in occurs, and as the voltage is decreased, the mirror will remain at 19 degrees until the voltage drops below the hold voltage of 124 V. The mirror will then return to an angle of 13.972 degrees. This is shown in figure 6.

4. CONCLUSION

The pull-in and hysterisis behavior of this vertical comb drive electrostatic actuator have been presented here. For this device, electrostatic pull-in occurs at 16.5 degrees and 132.71 V. The holding-voltage is determined to be 124 volts, and results in a small hysterisis loop in the device operation. Further analysis of the effects of these nonlinear behaviors on the dynamics of the device is currently under investigation.

5. ACKNOWLEDGEMENTS

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