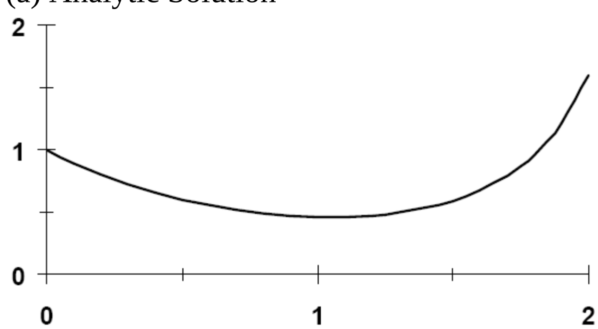
HOMEWORK ASSIGNMENT 8
EGM 3344
Numerical Integration of ODEs Concepts

Problems from Chapra book chapters 20 through 22

Problem	**Approach**	**Comments**
20.1	Matlab	Write simple Matlab programs to implement a single time step of Euler's method, the midpoint method, and the 4th order Runge-Kutta method. Then write a driver program that uses a `for` loop to iterate the calls to each method.
20.9	Matlab	Only do part (a). Convert the second order ODE into two first order ODEs to numerically integrate.
20.15	Matlab	Use the built-in Matlab function `ODE45` to perform the numerical integration.
21.3	Matlab	Only integrate from 2.0 to 2.5 sec.
21.6	Matlab	
21.11	Matlab	Use `ODE45` and `ODE23S` and note that `ODE45` will take much longer than `ODE23S` since this problem is numerically stiff.
22.1	Matlab	Only do parts (b) and (c). The (x, T) values for the analytic solution are provided in the answers below.

Answers

20.1
(a) Analytic Solution



(b) Euler with h = 0.5

x	*y*	*dy/dx*
0	1	-1.1
0.5	0.45	-0.3825
1	0.25875	-0.02588
1.5	0.245813	0.282684
2	0.387155	1.122749

Euler with h = 0.25

x	*y*	*dy/dx*
0	1	-1.1
0.25	0.725	-0.75219
0.5	0.536953	-0.45641
0.75	0.422851	-0.22728
1	0.36603	-0.0366
1.25	0.356879	0.165057
1.5	0.398143	0.457865
1.75	0.51261	1.005997
2	0.764109	2.215916

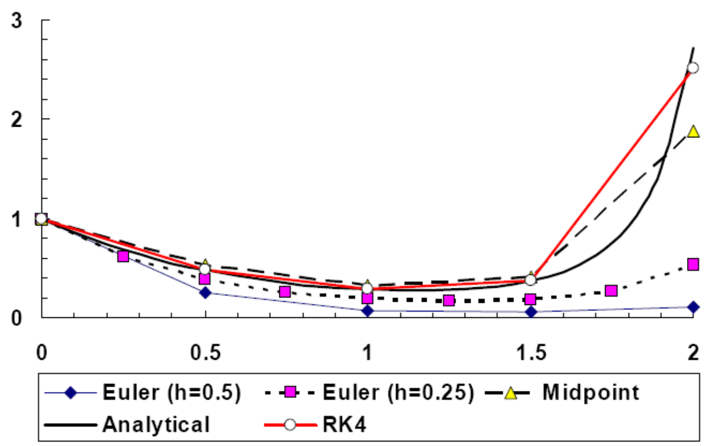
(c) Midpoint

t	y	dy/dt	t_m	y_m	dy_m/dt
0	**1**	-1.5	0.25	0.625	-0.92773
0.5	**0.536133**	-0.73718	0.75	0.351837	-0.37932
1	**0.346471**	-0.17324	1.25	0.303162	0.13737
1.5	**0.415156**	0.778417	1.75	0.60976	2.353292
2	**1.591802**				

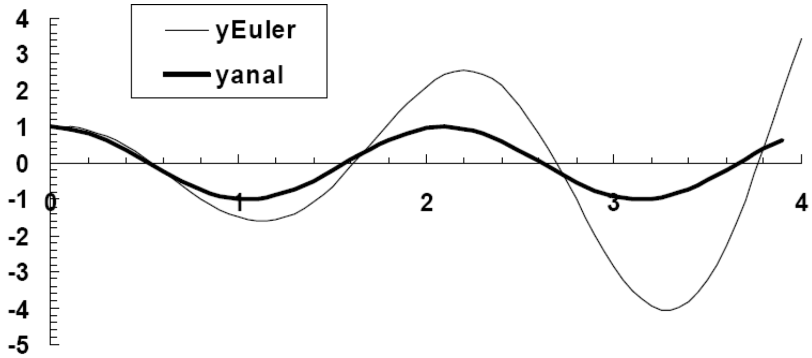
(d) RK4

t	y	k_1	t_m	y_m	k_2	t_m	y_m	k_3	t_e	y_e	k_4	ϕ
0	**1.0000**	-1.5000	0.25	0.6250	-0.9277	0.25	0.7681	-1.1401	0.5	0.4300	-0.5912	-1.0378
0.5	**0.4811**	-0.6615	0.75	0.3157	-0.3404	0.75	0.3960	-0.4269	1	0.2676	-0.1338	-0.3883
1	**0.2869**	-0.1435	1.25	0.2511	0.1138	1.25	0.3154	0.1429	1.5	0.3584	0.6720	0.1736
1.5	**0.3738**	0.7008	1.75	0.5489	2.1186	1.75	0.9034	3.4866	2	2.1170	13.7607	4.2786
2	**2.5131**											

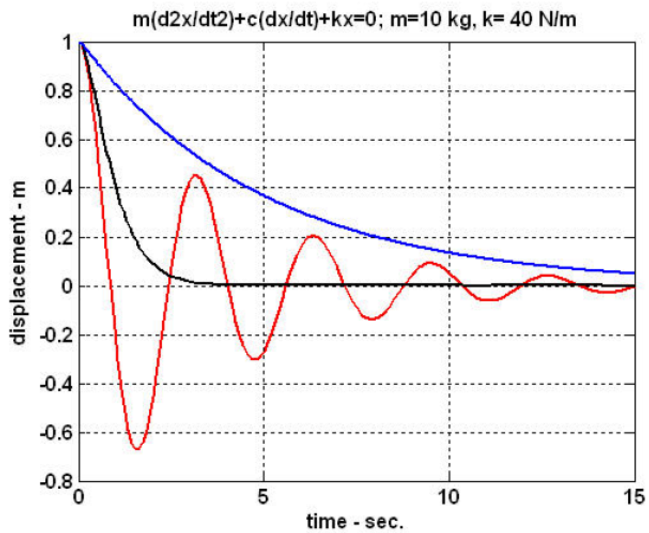
All solutions plotted on the same figure:



20.9



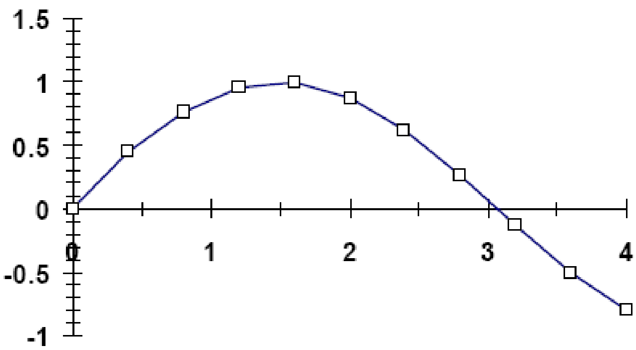
20.15



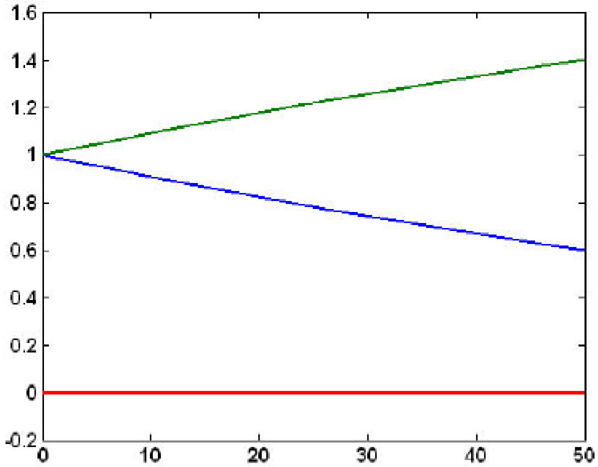
21.3

After iterating the correction, $y(2.5) = 3.271558$, $\varepsilon_a = 0.061$

21.6



21.11



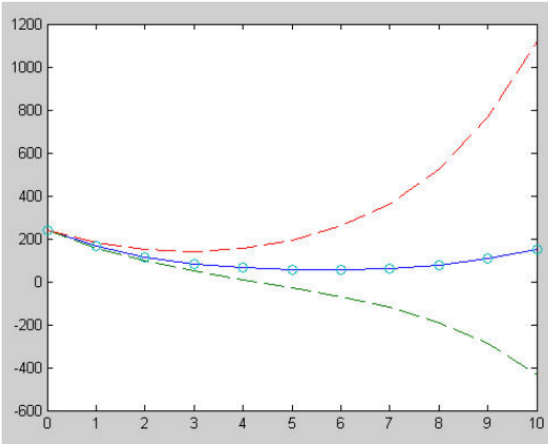
22.1

Analytic solution at specified *x* locations:

x	*T*
0	240
1	165.329
2	115.7689
3	83.79237
4	64.54254
5	55.09572
6	54.01709
7	61.1428
8	77.55515
9	105.7469
10	150

(b)

x	T	dT/dx
0	240.0000	-90.6147
1.0000	165.3278	-60.5889
2.0000	115.7683	-39.7664
3.0000	83.7921	-24.9838
4.0000	64.5424	-13.9958
5.0000	55.0957	-5.1334
6.0000	54.0171	2.9492
7.0000	61.1429	11.4799
8.0000	77.5553	21.7541
9.0000	105.7471	35.3325
10.0000	150.0000	54.2772



(c)

x	*T*	Analytical
0	240	240
1	165.7573	165.3290
2	116.3782	115.7689
3	84.4558	83.7924
4	65.2018	64.5425
5	55.7281	55.0957
6	54.6136	54.0171
7	61.6911	61.1428
8	78.0223	77.5552
9	106.0569	105.7469
10	150	150