Lecture 13

Optimization Analyses

EML 5595
Mechanics of the Human Locomotor System

Outline

- Optimization Motivation
- Optimization Concepts
- Journal Article Review

  Kautz and Hull (1995)

Outline

- Optimization Motivation

Why Use Optimization?

Many human movement problems involve biomechanical systems that possess more unknowns than available equations.

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Indeterminate Systems

Simple Example

$$3x + 2y = 10$$

What are *x* and *y*?

One solution: Choose $x = 2$, $y = 2$.

Another solution: Choose $x = 0$, $y = 5$.

Observation: Since we have more unknowns than equations, there is an infinite number of possible solutions. How do we choose one solution over another?

Simple Example

Principle: Choose some physically realistic condition to impose on *x* and *y*. For example, the magnitude of [*x*, *y*] must be as small as possible.

$$\min x^2 + y^2$$

subject to

$$3x + 2y = 10$$

Solution: *x* = 2.31, *y* = 1.54.

x	y	$x^2 + y^2$
2.31	1.54	7.69
2	2	8
0	5	25

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Optimization Elements

Design Variables: $\mathbf{x} = [x_1 \quad x_2 \quad \ldots \quad x_n]$

Cost function: $\min f(\mathbf{x})$

Constraints: $g_1(\mathbf{x}) = 0$

$g_2(\mathbf{x}) \leq 0$

$\longrightarrow$

$$\min f(\mathbf{x})$$

subject to

$$g_1(\mathbf{x}) = 0$$

$$g_2(\mathbf{x}) \leq 0$$

where $f(\mathbf{x})$, $g_1(\mathbf{x})$, and $g_2(\mathbf{x})$ can be linear or nonlinear functions of $\mathbf{x}$.

Optimization Methods

Method	Cost function	And/Or	Constraints
Linear Programming	Linear	And	Linear
Quadratic Programming	Quadratic	And	Linear
Nonlinear Programming	Nonlinear	Or	Nonlinear

Movement Optimization

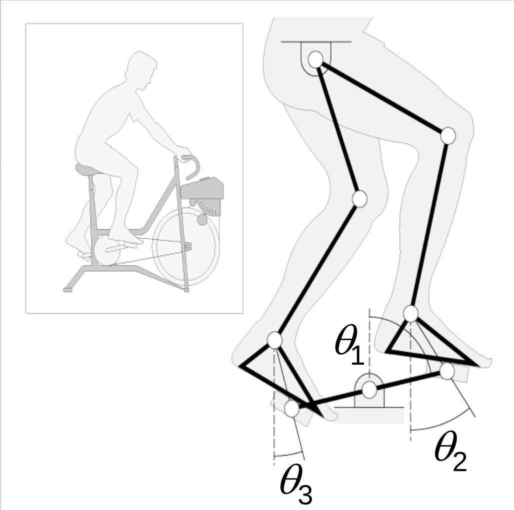
Quantities available for use as design variables in cost function and constraints:

- Experimental or simulated joint angles
- Experimental or simulated ground reactions
- System parameters (inertial, kinematic, muscle)
- Control nodal points (activations, forces, torques)
- Initial conditions (q's and qdot's)

Question: When should we use each of these quantities in the cost function? When should we use them in the constraints?

Problem Formulation

Define design variables, a cost function, and constraints to produce a symmetric pedaling motion that reproduces experimentally measured joint angles and pedal reaction forces.



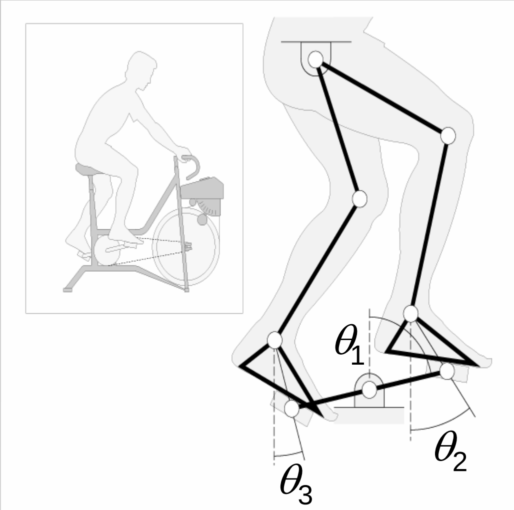
Helpful information: The cost function is reduced gradually with each step, while the constraints are met at each step along the way.

One Possibility

Design Variables:

- Torque control nodal points
- Initial conditions

Cost Function: Minimize the RMS error between the inverse dynamics and optimized joint torque controls



Constraints:

- Control symmetry
- Final conditions = initial conditions (terminal constraints), or
- RMS error in each joint angle < tol (path constraints)

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