Lecture 7

Inverse Dynamic Analyses

EML 5595
Mechanics of the Human Locomotor System

Outline

- Basic Inverse Dynamics Methods
- Advanced Inverse Dynamics Methods
- Journal Article Reviews

  Kuo (1998)

  Cahouet et al. (2002)

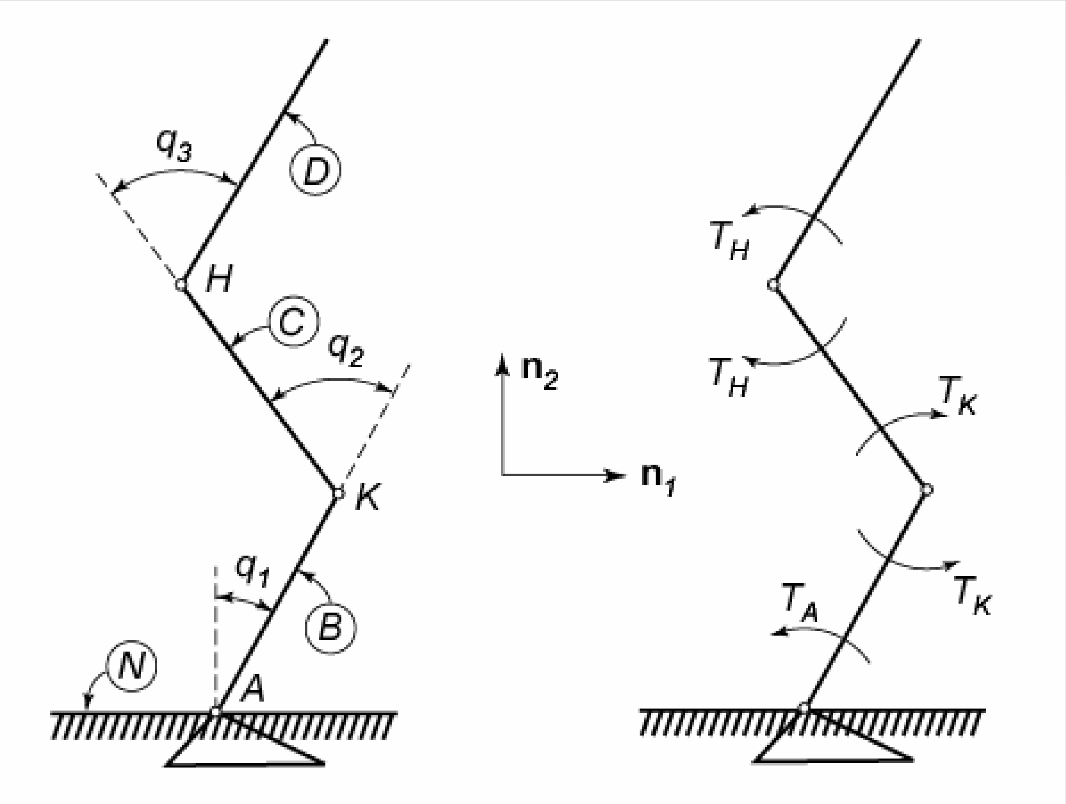
Outline

- Basic Inverse Dynamics Methods

Basic Methods

- Matrix Equations
- Bottom Up
- Top Down

Matrix Equations



Matrix Equations

$$\mathbf{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} = \mathbf{N}(\boldsymbol{\theta})\mathbf{T} + \mathbf{G}(\boldsymbol{\theta}) + \mathbf{V}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

$$\downarrow$$

$$\mathbf{N}(\boldsymbol{\theta})\mathbf{T} = \mathbf{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} - \mathbf{G}(\boldsymbol{\theta}) - \mathbf{V}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

Matrix Equations

$$\mathbf{M}(\theta)\ddot{\theta} = \mathbf{N}(\theta)\mathbf{T} + \mathbf{G}(\theta) + \mathbf{V}(\theta,\dot{\theta})$$

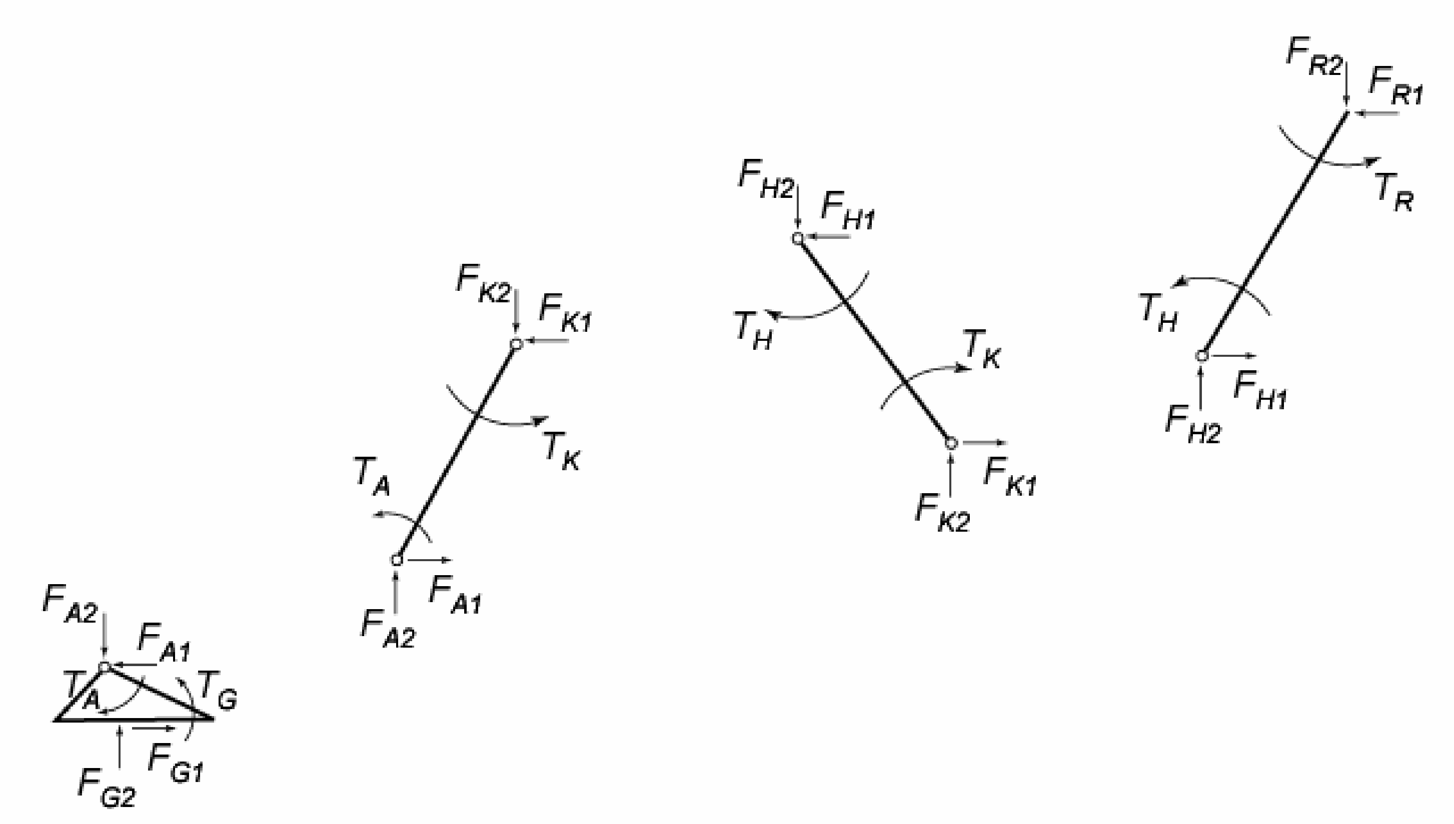
$$\downarrow$$

$$\underbrace{\mathbf{N}(\theta)}_{\mathbf{A}}\underbrace{\mathbf{T}}_{\mathbf{x}} = \underbrace{\mathbf{M}(\theta)\ddot{\theta} - \mathbf{G}(\theta) - \mathbf{V}(\theta,\dot{\theta})}_{\mathbf{b}}$$

$$\downarrow$$

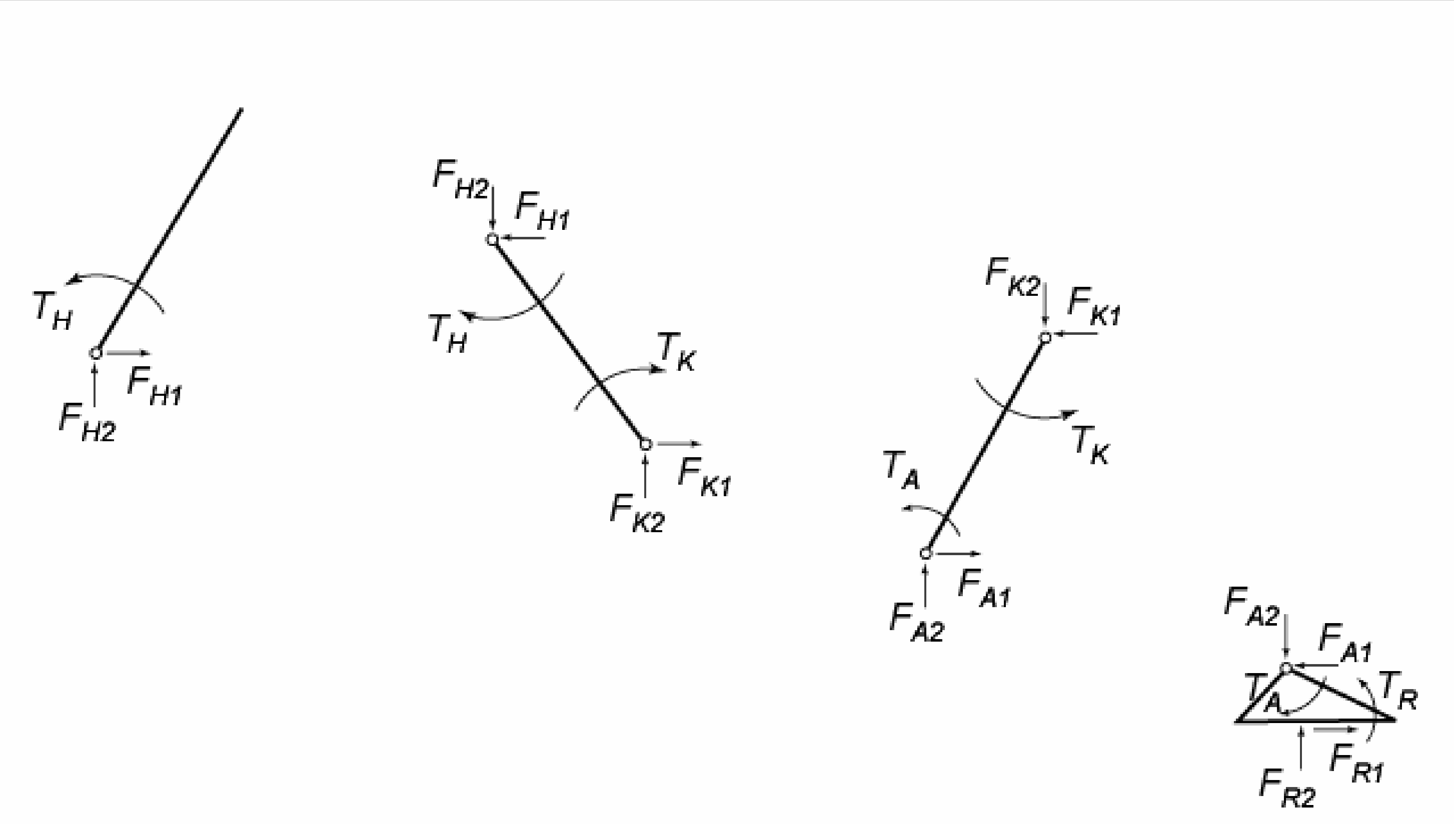
$$\mathbf{T} = \mathbf{N}^{-1}(\theta)\left[\mathbf{M}(\theta)\ddot{\theta} - \mathbf{G}(\theta) - \mathbf{V}(\theta,\dot{\theta})\right]$$

Bottom Up



F_{A2}
F_{A1}
T_A
T_G
F_{G1}
F_{G2}
F_{K2}
F_{K1}
T_K
T_A
F_{A1}
F_{A2}
F_{H2}
F_{H1}
T_H
T_K
F_{K1}
F_{K2}
F_{R2}
F_{R1}
T_R
T_H
F_{H1}
F_{H2}

Top Down



T_H
F_{H1}
F_{H2}
F_{H2}
F_{H1}
T_H
T_K
F_{K1}
F_{K2}
F_{K2}
F_{K1}
T_K
T_A
F_{A1}
F_{A2}
F_{A2}
F_{A1}
T_A
T_R
F_{R1}
F_{R2}

Summary

- Matrix Equations
  - Only requires kinematic data
  - Susceptible to kinematic errors in every segment

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  - Requires kinematic and ground reaction data
  - Each segment is only susceptible to kinematic errors in segments at or below it
  - Residual forces and torques at top of chain are not zero

Summary

- Matrix Equations
  - Only requires kinematic data
  - Susceptible to kinematic errors in every segment
- Bottom Up
  - Requires kinematic and ground reaction data
  - Each segment is only susceptible to kinematic errors in segments at or below it
  - Residual forces and torques at top of chain are not zero
- Top Down
  - Only requires kinematic data
  - Each segment is only susceptible to kinematic errors in segments at or above it
  - Residual forces and torques at bottom of chain do not match ground reactions

Question

If the inverse dynamics problem can be solved without ground reaction force and torque data, how can we make use of these additional (i.e., redundant) data to improve the accuracy of our calculations AND have the correct residual forces and torques at the ends of the chain?

Outline

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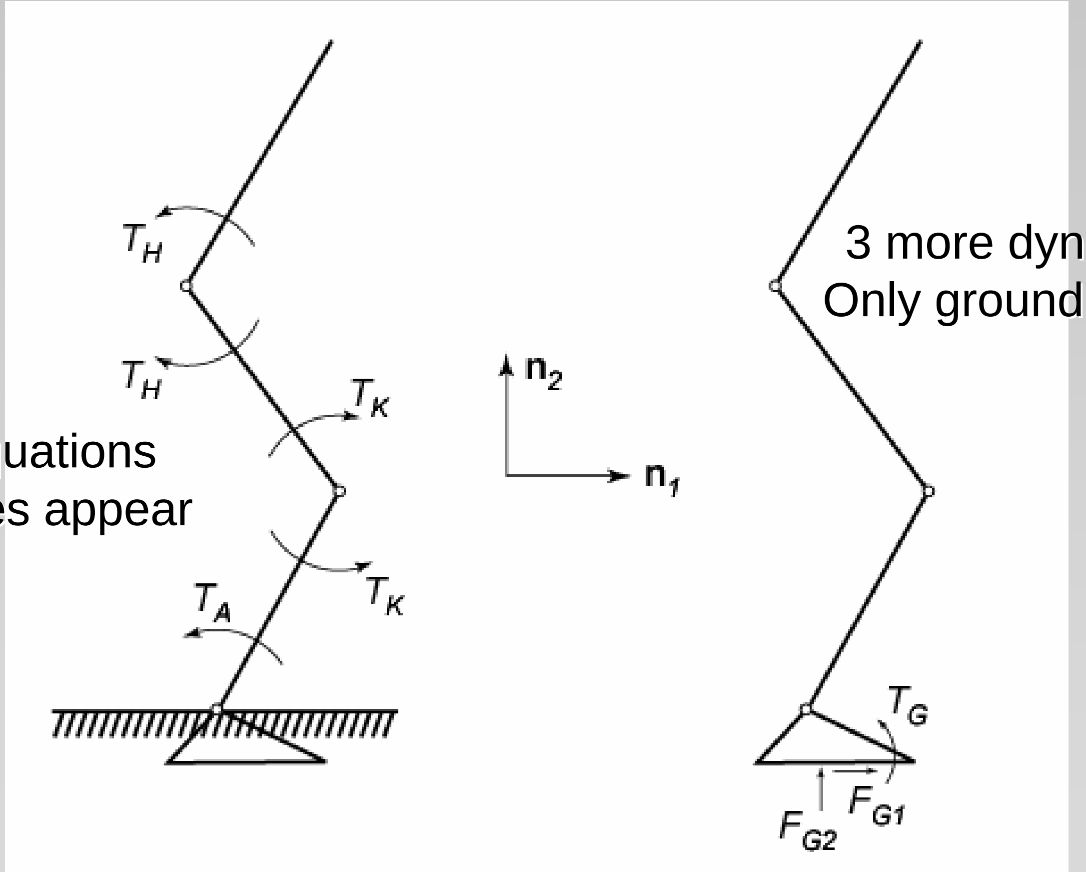
- Advanced Inverse Dynamics Methods

Observation



3 dynamics equations
Only joint torques appear

3 more dynamics equations
Only ground reactions appear

Key Concept

Combine the set of *n* dynamics equations containing joint torques with the 3 (planar) or 6 (spatial) additional dynamics equations containing ground reactions to produce a redundant set of *n*+3 or *n*+6 dynamics equations that can be solved for the *n* joint torques.

In this way, the additional 3 or 6 dynamics equations can compensate for errors in the estimated accelerations.

Solution Method

Linear Least Squares

$$\underbrace{\mathbf{A}}_{m\,x\,n}\ \underbrace{\mathbf{x}}_{n\,x\,1} = \underbrace{\mathbf{b}}_{m\,x\,1}$$

where m > n

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Advanced Method 1

- What is the goal of Kuo's method, and for which quantities does he solve after forming a redundant system of linear equations?

Advanced Method 1

- What is the goal of Kuo's method, and for which quantities does he solve after forming a redundant system of linear equations?
- Using the two dynamics equations presented by Kuo,

$$\mathbf{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} = \mathbf{N}(\boldsymbol{\theta})\mathbf{T} + \mathbf{G}(\boldsymbol{\theta}) + \mathbf{V}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

$$\mathbf{C}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} = \mathbf{R} + \mathbf{D}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

re-derive the redundant system of linear equations he uses to solve for the joint torques.

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Advanced Method 2

- What is the goal of Cahouet et al.'s method, and for which quantities do they solve after forming a redundant system of linear equations?

Advanced Method 2

- What is the goal of Cahouet et al.'s method, and for which quantities do they solve after forming a redundant system of linear equations?
- Using the dynamics equations presented by Cahouet et al.,

$$\mathbf{C}(\theta)\ddot{\theta} = \mathbf{R} + \mathbf{D}(\theta, \dot{\theta})$$

re-derive the redundant system of linear equations they use to solve for joint accelerations.

For Next Time

- Download and read the Gilchrist and Winter (1997) and the Panne and Lamouret (2002) articles on forward dynamics simulation.
- Bring any questions you have thus far on inverse or forward dynamics concepts.