A GENERALIZED ANALYTICAL JOINT CONTACT MODEL FOR DYNAMIC MUSCULOSKELETAL SIMULATIONS

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1. ABSTRACT

Dynamic musculoskeletal models are useful for predicting how muscles, ligaments, and bones interact under loaded, weight-bearing conditions. These predictions can be heavily influenced by the way in which the joints are modeled, with joint contact modeling being the weak link in the musculoskeletal modeling chain. This study presents a computationally efficient approach for modeling deformable contact between general articular surfaces possessing general material properties. The approach utilizes surrogate modeling, where a simple, computationally cheap model is fitted to the input-output relationships of a complex, computationally expensive model. The surrogate model's form matches an analytical Hertzian contact model except that the stiffness, interpenetration, and exponent are generalized to be functions of relative pose and applied axial load. During a dynamic simulation, where only the relative pose is known at any instant in time, all contact load components are determined from the surrogate contact model via an inverse procedure. A sample application is presented for a one cycle dynamic simulation of gait using a total knee replacement model constrained to planar motion. When the original elastic foundation contact model was replaced with a generalized analytical contact model, dynamic simulation time was reduced from 30 minutes to 20 seconds with no loss of accuracy. If extensible to three dimensions, the approach may permit inclusion of a large number of deformable joints in dynamic musculoskeletal models with minimal added computational cost or complexity.

2. INTRODUCTION

Musculoskeletal computer models are useful for estimating physiological quantities that cannot be measured experimentally (Buchanan *et al.*, 1996), and designing new medical devices and rehabilitation approaches (Neptune *et al.*, 2000). Lack of articular contact in musculoskeletal computer models can lead to inaccurate prediction of quantities influenced by the interactions between muscles, ligaments, and bones. However, unlike engineering joint models, articular contact models require repeated evaluation of surface geometry. In dynamic contact simulations, these geometry evaluations consume the vast majority of the CPU time.

Recently, the first dynamic knee simulation utilizing a surrogate contact modeling approach has been reported. This approach reduced the simulation time of one gait cycle

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to less than one and a half minutes (Lin *et al.*, 2005). Unfortunately, because the surrogate model was implemented using built-in functions, it was regenerated each time a value of contact force was needed by the numerical integrator. In addition, the technique used in that work was only suitable for joints constrained to planar motion.

This study proposes a generalized surrogate modeling technique to speed up dynamic musculoskeletal simulations incorporating articular surface contact. This approach involves using knowledge of contact mechanics to fit a computationally-cheap surrogate model to data points generated by a computationally-costly contact model. The goal was to determine whether a dynamic simulation performed with the generalized surrogate contact model could closely reproduce the planar motions and contact forces produced by an elastic foundation (EF) contact model (i.e., the computationally-costly contact model) but with substantially reduced computational cost. The proposed surrogate contact model can be viewed as a generalized Hertzian contact model where the stiffness and exponent in the model are allowed to vary.

3. METHODS

A 3D artificial knee contact model (Fig. 1) created from an Osteonics 7000 cruciate-retaining knee implant (Stryker Howmedica Osteonics, Inc, Allendale, NJ) was constructed to evaluate the proposed surrogate modeling technique. The tibial insert of this contact model was fixed to ground and the femoral component was connected to the tibia via a six degree-of-freedom (DOF) joint. A fine element grid of 100 x 100 was employed on the medial and lateral sides of the tibial insert. EF based contact code was incorporated into the contact model and used to solve for the medial and lateral contact conditions as a function of the six DOFs between tibial insert and femoral component. Details of the model creation process and EF theory can be found in Bei and Fregly (2004). In this paper, we simplified the model to three DOFs, anterior-posterior (AP) translation x_t, superior-inferior (SI) translation y_t, and flexion z_r, by constraining the medial-lateral translation, internal-external rotation, and varus-valgus rotation to be zero. Dynamic equations were derived for the femoral component moving in the sagittal plane with respect to a fixed tibial insert. Sagittal plane rotation (i.e., z_r) was prescribed while x_t and y_t were predicted by numerically integrating the equations of motion using the stiff solver ode15s available in Matlab (The Mathworks, Natick, MA). The prescribed flexion angle and applied loads in the x_t and y_t directions were taken from a planar version of a dynamic simulation performed in a previous study using the same implant mode (Fregly *et al*., 2005).

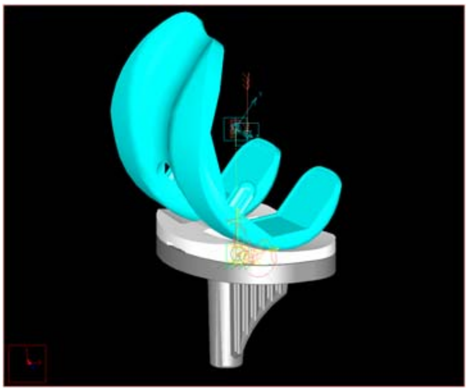

Figure 1: 3 DOF artificial knee elastic foundation contact model.

AP (F_x) and SI (F_y) contact forces applied to the femoral component and tibial insert (equal and opposite) were included in the dynamic simulation using two different contact models. The first one was an EF contact model and the second one was a surrogate contact model created from sample points generated by the EF contact model. The goal was to determine whether a dynamic simulation performed with the surrogate contact model could closely reproduce the SI and AP motions and contact forces produced by the EF contact model but with substantially reduced computational cost. For both contact models, the dynamic simulation was performed on a 2.8 GHz Pentium IV PC.

Development of the surrogate contact model involved four steps: 1) Design of experiments, 2) Computational experiments, 3) Surrogate model selection, and 4) Surrogate model evaluation (Fig. 2). Design of experiments is a statistical method for determining which locations in the input variable space (x_t, y_t, and z_r) should be sampled for predicting outputs of interest from the computationally-costly model (F_x and F_y from EF contact model). To maximize the quality of the resulting surrogate model fit while minimizing the number of sample points, we choose the Latin hypercube sampling (LHS) scheme, which places at most one sample point in each row and column of the multi-dimensional design space. Since contact forces are highly sensitive to changes in SI translation, LHS was used to specify 150 pairs of x_t and z_r values whose extremes were determined by the limits of realistic motions obtained from in vivo fluoroscopic measurements (Fregly *et al.*, 2005). At each location on the 150 LHS points, six sample points were generated by performing a static analysis with the EF contact model for applied F_y varying from 10 to 3010 N in 500 N increments. Initial attempts to use statistical sampling methods that varied x_t, y_t, and z_r together proved unsuccessful since random sampling in the y_t direction produced only a sparse number of points in contact.

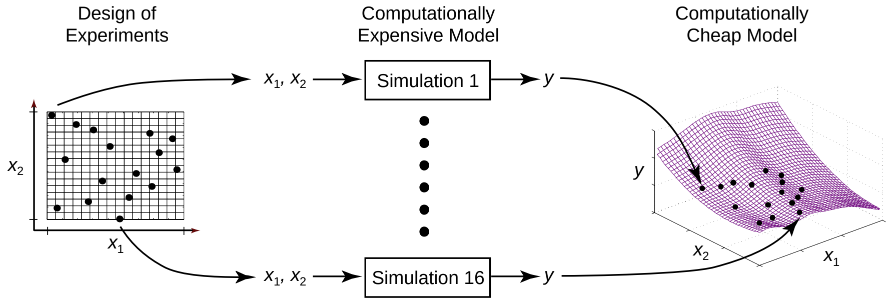

Figure 2: Overview of the steps involved in the surrogate model creation process. Sample points in design space are determined using statistical techniques from design of experiments (left). An analysis is performed with the computationally costly model at each of the specified design points (middle). The input-output relationship of interest is fitted with a computationally cheap surrogate model (essentially a multi-dimensional surface) (right) which can then be used in place of the original computationally costly model.

To fit the variation of Fy through six different contact loads, we employed the structure of a Hertzian contact model. For EF contact problems, contact forces increase monotonically and nonlinearly with interpenetration (Johnson, 1985). Therefore, we developed a generalized contact model of the form

$$\hat{F}_y = k_y \bar{y}_t^{n_y} \qquad [1]$$

where $\hat{F}_y^i$ is the predicted SI contact force, k_y is the contact stiffness, $\bar{y}_t = y_t^{\max} - y_t$ is the SI translation between the maximum (i.e., the contact initiation) and current y_t, and

n_y is the contact exponent. At each location on the 150 LHS points, the non-linear least squares optimization was performed with the Matlab Optimization Toolbox to seek the optimized values of $y_t^{\max}$, k_y, and n_y. Since the Hertizan contact model is suitable only for normal contact force, Eq. [2] is needed for predicting F_x.

$$\hat{F}_x = ratio \cdot \hat{F}_y \qquad [2]$$

where $\hat{F}_x$ is the predicted AP contact force, *ratio* is the average of six quotients obtained from six known values of F_x divided by F_y.

After 1050 static analyses, Kriging (Sacks *et al.*, 1989) model and polynomial with Fourier harmonics model were used to fit k_y, n_y, $y_t^{\max}$, and *ratio* as functions of x_t and z_r. Kriging model is the combination of a polynomial model with a correlation function. After evaluating several different combinations, a quartic polynomial plus the Gaussian correlation function were selected to construct Kriging model while poly-Fourier model was modeled by a cubic polynomial with one Fourier harmonic. Each time the dynamic simulation required a value for F_x and F_y, the surrogate contact models used the current values of x_t, y_t, and z_r to perform the following calculations. First, new values of k_y, n_y, $y_t^{\max}$, and *ratio* were calculated from their corresponding surrogate models using x_t and z_r as inputs. Next, $\bar{y}_t$ was calculated for the current value of y_t. Finally, F_y was calculated from Eq. [1], which was used to calculate F_x using Eq. [2].

4. RESULTS

The dynamic simulation that used the generalized surrogate contact models closely reproduced the motions and loads predicted with the EF contact model (Fig. 3). However, motion and force were matched more closely in the SI direction than in the AP direction. Furthermore, F_x predictions from the Kriging model were slightly more accurate than those from the poly-Fourier model. While the dynamic simulation using the EF contact model required 74 minutes of CPU time, the simulation using surrogate contact models required less than 20 seconds.

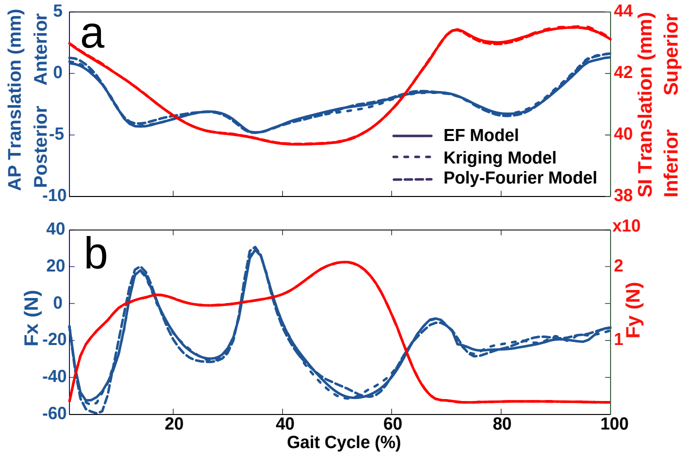

Figure 3: Comparison of motions and loads predicted by the EF and two surrogate contact models. (a) AP (blue) and SI (red) translation. (b) Fx (blue) and Fy (red) forces.

5. DISCUSSION

Using a surrogate model to replace the original articular contact model possesses several advantages. First, generation of sample points for surrogate model construction requires repeated static rather than dynamic analyses. Moreover, instead of distributing the sampling points uniformly in the design space LHS sampling scheme was chosen for its practical application in high-dimensional design space problem. Second, the density of the contact element grid only affects the surrogate model creation process and has no effect on surrogate model run-time cost. Though a denser contact element grid will produce more accurate results with the EF contact model, the increase in computational cost can be substantial. For example, a contact element grid of 100 x 100 requires 74 minutes of CPU to perform the same dynamic simulation. Third, the design variables used in the optimization are not limited to (and need not be) the inputs to the surrogate model. Furthermore, use of large numbers of design variables, which is common in human movement optimizations, does not create difficult high-dimensional surrogate model fitting problems, since the optimization design variables can be independent of the surrogate model inputs. Fourth, the use of design optimization which usually requires hundreds or even thousand of repeated dynamic simulations can be efficiently realized using surrogate modeling approach. Finally, the surrogate model can br embedded within any large dynamic model. Thus, the system to be optimized can be changed without the need to generate new surrogate models, facilitating optimizations involving full-body musculoskeletal models.

This paper has presented a generalized surrogate modeling technique for efficient dynamic simulation of anatomic joints. The technique is demonstrated using a dynamic simulation of a sample knee replacement constrained to planar motion. Two different surrogate contact models produced comparable accuracy with orders of magnitude improvement in computational speed. The primary limitations of this approach are that the calculation of F_x is dependent on the values of *ratio* and F_y computed from Eq. [1]. Independent calculation of F_x and the extension of this approach to high-dimensional problems are topics for future research efforts.

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