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A METHOD FOR INFERRING THE OPTIMIZATION COST FUNCTION OF EXPERIMENTALLY OBSERVED MOTOR STRATEGIES

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ABSTRACT

We propose a computational procedure for inferring the cost functions that, according to the Principle of Optimality, underlie experimentally observed motor strategies. This work tries to overcome the need to hypothesize the cost functions, extracting this non-directly observable information from experimental data. Optimality criteria of observed motor tasks are here indirectly derived using: a) a mathematical model of the bio-system; and b) a parametric mathematical model of the possible cost functions, i.e. a search space constructed in such a way as to presumably contain the unknown function that was used by the bio-system in the given motor task of interest. The cost function that best matches the experimental data is identified within the search space by solving a nested optimization problem. This problem can be recast as a non-linear programming problem and therefore solved using standard techniques. The proposed methodology is tested on representative examples.

INTRODUCTION

The Principle of Optimality [7] states that the neuro-musculoskeletal system is a *teleonomic* system, i.e. a goal-oriented system that behaves so as to minimize certain cost functions (performance indices) [6]. If these cost functions were known, the inverse neuro-musculoskeletal problem (finding the unknown, redundant controls) could be formulated as an optimal control problem for a suitable mathematical model of the bio-system. Unfortunately, the cost function that is at the heart of the Principle of Optimality can not be directly measured and needs to be hypothesized.

In this work, we propose a numerical procedure that allows one to *indirectly determine* the optimization cost function from available experimental data. The idea is to construct a model of the possible cost functions and, within this search space, determine the one that predicts a motion and the associated controls of the bio-system that are as close as possible to the observed ones. In other words, the mathematical model of the system becomes now a device that is used for determining a non-directly observable and hidden quantity from experimental data. The determination is indirect, in the sense that it is based on both a model of the bio-system and on a model of the cost function.

The paper has been organized according to the following plan. At first we introduce the mathematical models of the neuro-musculoskeletal system and we describe the Principle of Optimality. Next, we formulate our proposed methodology for inferring the cost function from experimental data. The formulation leads to the solution of a nested discrete parameter optimization problem, i.e. a (outer) parameter optimization problem that has among its constraints another (inner) parameter optimization problem. The resulting nested optimization problems is solved by expressing the necessary conditions for optimality of the inner problem, and by appending them as additional constraints to the outer optimization problem. This yields a standard parameter optimization problem that can be solved efficiently using classical techniques. Finally, the proposed methodology is tested on a representative problem. More details on the procedures here described and further examples are given in Reference [4].

THE PRINCIPLE OF OPTIMALITY IN BIOMECHANICS

We consider a neuro-musculoskeletal system $\mathcal{S}$ and its mathematical idealization $\mathcal{M}$. Model $\mathcal{M}$ is in general a multibody system that includes rigid bodies with their inertial parameters, joints, muscles with their mechanical and physiological properties, interactional forces with the environment, and other components as required for the accurate representation of the real bio-system $\mathcal{S}$. The governing equations for system $\mathcal{M}$ can be written as

$$\dot{\boldsymbol{y}} - \boldsymbol{f}(\boldsymbol{y},\boldsymbol{u}) = 0, \tag{1}$$

where $\boldsymbol{y} \in \mathbb{R}^{n_y}$ are the state variables, $\boldsymbol{u} \in \mathbb{R}^{n_u}$ are the controls, and $\boldsymbol{f} : \mathbb{R}^{n_y} \times \mathbb{R}^{n_u} \rightarrow \mathbb{R}^{n_y}$. This dynamics problem is defined over the domain $\Omega := (T_0, T) \subset \mathbb{R}$ with boundary $\Gamma := \{T_0, T\}$, $t \in \Omega$, t being the time and the symbol $\dot{(\cdot)}$ indicating derivatives with respect to t, i.e. $\dot{(\cdot)} := \mathrm{d}(\cdot)/\mathrm{d}t$. For a static problem, the equations of equilibrium can be written as

$$\boldsymbol{f}(\boldsymbol{y},\boldsymbol{u}) = 0. \tag{2}$$

When the system contains closed loops or a redundant coordinate formulation is used, the multibody governing equations of motion will be in the form of Differential Algebraic Equations (DAE), rather than in the Ordinary Differential Equation (ODE) form of Equation (1), and they will write

$$\dot{\boldsymbol{y}} - \boldsymbol{f}(\boldsymbol{y},\boldsymbol{u},\boldsymbol{z}) = 0, \tag{3}$$
$$\boldsymbol{c}(\boldsymbol{y}) = 0, \tag{4}$$

where $\boldsymbol{c} : \mathbb{R}^{n_y} \rightarrow \mathbb{R}^{n_z}$ represent the multibody constraint equations, enforced by the Lagrange multipliers $\boldsymbol{z} \in \mathbb{R}^{n_z}$. The formulation described here is valid also in the DAE case, but for notational simplicity we restrict our attention to the sole ODE case (1) in the following.

The values of the states and of the controls that are measured on system $\mathcal{S}$ for experiment $\mathcal{E}$ are noted $\boldsymbol{y}^{\mathcal{E}}$ and $\boldsymbol{u}^{\mathcal{E}}$, respectively. The compatibility of the mathematical model $\mathcal{M}$ with the system $\mathcal{S}$ in the case $\mathcal{E}$ can be measured by evaluating the residuals of the governing equations, by replacing the system states and controls with the corresponding measured quantities:

$$\boldsymbol{\varepsilon} := \dot{\boldsymbol{y}}^{\mathcal{E}} - \boldsymbol{f}(\boldsymbol{y}^{\mathcal{E}}, \boldsymbol{u}^{\mathcal{E}}). \tag{5}$$

An analogous expression can be obtained in the static case. The residuals $\boldsymbol{\varepsilon}$ differ from zero because: a) the experimental quantities are affected by measurement errors, which are typically particularly relevant for the controls $\boldsymbol{u}^{\mathcal{E}}$, such as muscle or motoneuron activations; b) the mathematical model $\mathcal{M}$ is only an approximation of the real bio-system $\mathcal{S}$. Assuming that the experimental errors can be kept sufficiently small using suitable testing procedures and equipment, the residuals $\boldsymbol{\varepsilon}$ can be used to measure the *fidelity* of the mathematical model $\mathcal{M}$ to the system $\mathcal{S}$ for the specific experiment $\mathcal{E}$ corresponding to the measured values $\boldsymbol{y}^{\mathcal{E}}$ and $\boldsymbol{u}^{\mathcal{E}}$.

Consider now a model $\mathcal{M}$ that is of acceptable fidelity for experiment $\mathcal{E}$, i.e. a model such that $\boldsymbol{\varepsilon}$ is sufficiently small in some suitable norm, for example the infinity norm $||\cdot||_\infty$. The Principle of Optimality states that there is a cost function $J^*(\boldsymbol{y},\boldsymbol{u})$ such that $||\boldsymbol{\varepsilon}_y||_\infty$, $||\boldsymbol{\varepsilon}_u||_\infty$ are *small*, where

$$\boldsymbol{\varepsilon}_y := \boldsymbol{y}^* - \boldsymbol{y}^{\mathcal{E}}, \tag{6}$$
$$\boldsymbol{\varepsilon}_u := \boldsymbol{u}^* - \boldsymbol{u}^{\mathcal{E}}, \tag{7}$$

when $\boldsymbol{y}^*$ and $\boldsymbol{u}^*$ are the solutions of the optimal control problem with cost function J^* for system $\mathcal{M}$:

$$\begin{aligned} &\min_{\boldsymbol{y},\boldsymbol{u}} \; J^*(\boldsymbol{y},\boldsymbol{u}) \\ &\text{s.t.: } \dot{\boldsymbol{y}} - \boldsymbol{f}(\boldsymbol{y},\boldsymbol{u}) = 0, \\ &\qquad \boldsymbol{g}(\boldsymbol{y},\boldsymbol{u}) \leq 0, \end{aligned} \tag{8}$$

with a similar expression for the static case.

The vector of constraints $\boldsymbol{g}(\boldsymbol{y},\boldsymbol{u}) := (\boldsymbol{g}_e(\boldsymbol{y},\boldsymbol{u})^T, \boldsymbol{g}_i(\boldsymbol{y},\boldsymbol{u})^T)^T$, $\boldsymbol{g} : \mathbb{R}^{n_y} \times \mathbb{R}^{n_u} \rightarrow \mathbb{R}^{n_c}$, collectively represent all equality ($\boldsymbol{g}_e(\boldsymbol{y},\boldsymbol{u}) = 0$) and inequality ($\boldsymbol{g}_i(\boldsymbol{y},\boldsymbol{u}) \leq 0$) constraints that must be satisfied by the solution, including possible non-linear constraints and bounds on states and controls, and boundary (initial and/or final) conditions on the states.

The Principle of Optimality states that the solution of an optimal control problem on model $\mathcal{M}$ with cost function J^* will closely match the experimental measurements, the (small) errors $\boldsymbol{\varepsilon}_y$, $\boldsymbol{\varepsilon}_u$ being only due to the approximate fidelity (Equation (5)) of $\mathcal{M}$ to $\mathcal{S}$ for case $\mathcal{E}$. For a model of infinite fidelity, the results $\boldsymbol{y}^*$, $\boldsymbol{u}^*$ of the optimal control problem would exactly match the experimental data $\boldsymbol{y}^{\mathcal{E}}$, $\boldsymbol{u}^{\mathcal{E}}$.

Unfortunately, the functional form of the cost function $J^*(\boldsymbol{y},\boldsymbol{u})$ is unknown, in general. The classical approach used in biomechanics to overcome this difficulty has been to: a) hypothesize a functional form for J^*; b) solve problem (8); c) check the errors in the computed states and controls with respect to the experimental measurements, Equations (6) and (7). This is a *trial and error* process, with no guarantee of success. In the following, we propose a numerical procedure for *inferring* J^* from the data of experiment $\mathcal{E}$.

COMPUTING COST FUNCTIONS FROM EXPERIMENTAL DATA

In this section we discuss a possible *constructive* process for computing an approximation to J^*. The unknown cost function belongs to some infinite dimensional space of cost functions,

$$J^*(\boldsymbol{y},\boldsymbol{u}) \in C_J(\boldsymbol{y},\boldsymbol{u}). \tag{9}$$

C_J is the space of all possible cost functions that will make problem (8) solvable. The idea is now to construct a local approximation to C_J, here termed a *search space of cost functions* $S_J := \{J(\boldsymbol{y},\boldsymbol{u};\boldsymbol{p}) | \boldsymbol{p} \in \mathbb{R}^{n_p}\}$, where $\boldsymbol{p}$ are unknown parameters. Clearly, S_J does not span the whole space of cost functions, i.e.

$$S_J \subset C_J; \tag{10}$$

however, S_J is constructed based on insight into the physical processes governing the biomechanics problem, in such a way that

$$J^*(\boldsymbol{y},\boldsymbol{u}) \approx J(\boldsymbol{y},\boldsymbol{u};\boldsymbol{p}^*) \in S_J, \tag{11}$$

for some yet to be determined value of the parameters $\boldsymbol{p}^*$.

Example: Consider for simplicity a static problem, with system governing equations (2) for model $\mathcal{M}$. The cost function space C_J is composed of all C^1 continuous functions of $\boldsymbol{y}$ and $\boldsymbol{u}$. For illustrative purposes, assume (based on suggestions in previous publications) that cost functions are expressed in terms of powers greater than one of neural activations a_i or powers of muscle forces f_i, for the generic muscle i in model $\mathcal{M}$. Hence a possible choice for the search space could be

$$J(\boldsymbol{y},\boldsymbol{u};\boldsymbol{p}) := \sum_{k=2}^{N_k} p_{a,k} \sum_{i=1}^{N_m} a_i^k + \sum_{k=2}^{N_k} p_{f,k} \sum_{i=1}^{N_m} f_i^k, \tag{12}$$

where N_k is a maximum exponent value, say for example $N_k = 4$, and N_m is the number of muscles in the model. In this case, the unknown parameters $p_{a,k}$, $k = 2,\dots,N_k$, represent the level of participation of the quadratic, cubic, etc. powers of the neuro activations to the cost function. Similarly, the unknown parameters $p_{f,k}$, $k = 2,\dots,N_k$, represent the level of participation of the corresponding functions of the muscle forces. Therefore, we would define in this case $\boldsymbol{p} := (\dots, p_{a,k}, \dots, p_{f,k}, \dots)^T$, $k = 2,\dots,N_k$. Note that a normalization condition needs to be enforced among the parameters with the present choice of the cost function search space, namely

$$\sum_{k=2}^{N_k} (p_{a,k} + p_{f,k}) = 1, \tag{13}$$

since only the relative weights among the various terms can be determined. Furthermore, the weights must be non-negative:

$$p_{a,k} \geq 0, \quad k = 2,\dots,N_k, \tag{14}$$

$$p_{f,k} \geq 0, \quad k = 2,\dots,N_k. \tag{15}$$

By constructing the search space, one transforms the problem of identifying J^* into the problem of finding the parameters $\boldsymbol{p}^*$ that will give the best matching with the experimental data, i.e. that will minimize the errors given by Equations (6) and (7).

The cost function inference problem for dynamics can be formulated as the following nested optimization problem:

$$\begin{aligned} &\min_{\boldsymbol{p}} \frac{\int_\Omega \left(||\boldsymbol{y} - \boldsymbol{y}^{\mathcal{E}}|| + ||\boldsymbol{u} - \boldsymbol{u}^{\mathcal{E}}|| \right) \mathrm{d}t}{\int_\Omega \left(||\boldsymbol{y}^{\mathcal{E}}|| + ||\boldsymbol{u}^{\mathcal{E}}|| \right) \mathrm{d}t} \\ &\text{s.t.: } \boldsymbol{h}(\boldsymbol{p}) \leq 0, \\ &\qquad \min_{\boldsymbol{y},\boldsymbol{u}} J(\boldsymbol{y},\boldsymbol{u};\boldsymbol{p}) \\ &\qquad \text{s.t.: } \dot{\boldsymbol{y}} - \boldsymbol{f}(\boldsymbol{y},\boldsymbol{u}) = 0, \\ &\qquad\qquad \boldsymbol{g}(\boldsymbol{y},\boldsymbol{u}) \leq 0. \end{aligned} \tag{16}$$

The cost function $\int_\Omega \left(||\boldsymbol{y} - \boldsymbol{y}^{\mathcal{E}}|| + ||\boldsymbol{u} - \boldsymbol{u}^{\mathcal{E}}|| \right) \mathrm{d}t / \int_\Omega \left(||\boldsymbol{y}^{\mathcal{E}}|| + ||\boldsymbol{u}^{\mathcal{E}}|| \right) \mathrm{d}t$ measures the normalized error between the numerical solution and the experimental data. The norm $||\cdot||$ will be a dimensionally consistent norm, or simply the 2-norm if the states and controls are non-dimensionalized [4]. The error measured by this cost function will be a minimum for the solution $\boldsymbol{y}^*$, $\boldsymbol{u}^*$, $\boldsymbol{p}^*$ of problem (16).

The optimization process is subjected to two constraints. The first one, $\boldsymbol{h}(\boldsymbol{p}) \leq 0$, imposes possible equality or inequality conditions on the parameters $\boldsymbol{p}$, such as the normalization and non-negativity of participation factors in the previously discussed example. The second constraint is an optimal control problem, that corresponds to problem (8) for a varying cost that is a function of $\boldsymbol{p}$.

Problem (16) represents the formulation that is here proposed for inferring the cost function of a dynamics problem in biomechanics. Given a functional form for the search space and experimental measurements, the solution to this problem will give the value of the parameters $\boldsymbol{p}$ that yield a state and control solution that is as close as possible to the measured data.

The solution to this problem converges to the exact solution. In fact, consider the case of a model $\mathcal{M}$ of infinite fidelity and assume that $J^*(\boldsymbol{y},\boldsymbol{u}) \in S_J$. Then at the solution the value of the error $\int_\Omega \left(||\boldsymbol{y} - \boldsymbol{y}^{\mathcal{E}}|| + ||\boldsymbol{u} - \boldsymbol{u}^{\mathcal{E}}|| \right) \mathrm{d}t$ will be zero, i.e. the state and control errors will be null, and furthermore the cost will have been identified as $J(\boldsymbol{y}^*,\boldsymbol{u}^*;\boldsymbol{p}^*) = J^*(\boldsymbol{y}^*,\boldsymbol{u}^*) + J_0$, J_0 being an undeterminable (and irrelevant) constant.

Problem (16) is a *nested optimal control problem*, which can be transformed into a nested parameter optimization problem using the direct transcription method [2, 4]. The basic idea is to discretize the governing dynamic equations (1) using some numerical methods on a suitable grid of the problem domain. The discretization process transforms the governing ODEs (or DAEs) into algebraic equations, whose unknown parameters are the values of the states and of the controls on the grid. Next, the cost function and the remaining problem equality and inequality constraints are expressed in terms of the discrete state and control values.

The cost function inference problem for statics can be formulated as the following nested optimization problem:

$$\begin{aligned} \min_{\boldsymbol{p}} \; & \frac{\sum_{i=1}^{N_c}\left(||\boldsymbol{y}_i - \boldsymbol{y}_i^{\mathcal{E}}|| + ||\boldsymbol{u}_i - \boldsymbol{u}_i^{\mathcal{E}}||\right)}{\sum_{i=1}^{N_c}\left(||\boldsymbol{y}_i^{\mathcal{E}}|| + ||\boldsymbol{u}_i^{\mathcal{E}}||\right)} \\ \text{s.t.:}\; & \boldsymbol{h}(\boldsymbol{p}) \le 0, \\ & \min_{\boldsymbol{y}_i,\boldsymbol{u}_i} \sum_{i=1}^{N_c} J(\boldsymbol{y}_i,\boldsymbol{u}_i;\boldsymbol{p}), \\ & \text{s.t.:}\; \boldsymbol{f}(\boldsymbol{y}_i,\boldsymbol{u}_i) = 0, \quad (i = 1,\ldots,N_c), \\ & \qquad \boldsymbol{g}(\boldsymbol{y}_i,\boldsymbol{u}_i) \le 0, \quad (i = 1,\ldots,N_c), \end{aligned} \tag{17}$$

where N_c represents the number of test cases considered in experiment $\mathcal{E}$ (corresponding, for example, to N_c different values of a joint angle or some other problem parameter). Here again the cost function inference leads to an optimization problem subjected to two constraints: possible conditions on the unknown parameters, and an inner optimization problem with varying cost J. Hence, even in this case we are lead to the solution of a nested NLP problem, whose solution will yield the best available estimate of the cost function within the search space with respect to the experimental data.

The cost function inference problem can be formulated as a nested NLP problem both for statics and dynamics, in the latter case after direct transcription. A general formulation that embraces both cases can be obtained by defining a vector of unknown discrete parameters, labelled $\boldsymbol{x}$. In the dynamics case, the unknown parameter vector is composed of the discrete state and control values on the computational grid. In the static case, the same vector is defined as $\boldsymbol{x} := (\ldots,\boldsymbol{y}_i^T,\ldots,\boldsymbol{u}_i^T,\ldots)^T$, $i = 1,\ldots,N_c$, and it is composed of the state and control values for each test case considered in experiment $\mathcal{E}$. Similarly, a vector of experimental observations is constructed and labeled $\boldsymbol{x}^{\mathcal{E}}$. With this notation, the nested NLP problem can be written as

$$\begin{aligned} \min_{\boldsymbol{p}} \; & \frac{||\boldsymbol{x} - \boldsymbol{x}^{\mathcal{E}}||}{||\boldsymbol{x}^{\mathcal{E}}||} \\ \text{s.t.:}\; & \boldsymbol{h}(\boldsymbol{p}) \le 0, \\ & \min_{\boldsymbol{x}} J(\boldsymbol{x};\boldsymbol{p}) \\ & \text{s.t.:}\; \boldsymbol{f}(\boldsymbol{x}) = 0, \\ & \qquad \boldsymbol{g}(\boldsymbol{x}) \le 0, \end{aligned} \tag{18}$$

where $\boldsymbol{f}(\boldsymbol{x}) = 0$ now represents the discrete (respectively, discretized) equations of static (respectively, dynamic) equilibrium, and $\boldsymbol{g}(\boldsymbol{x}) \le 0$ are the associated equality and inequality constraints.

In this work, the nested NLP problem (18) is solved by first explicitly deriving the optimality conditions of the inner NLP problem, and appending these additional conditions to the outer problem as constraints. This results into a standard NLP problem, that can be solved using Sequential Quadratic Programming (SQP) [3]. More details on this procedure are given in Reference [4].

NUMERICAL EXAMPLE

We consider the human lower limb problem described in Reference [9], and illustrated in Figure 1. During the experiments [10], the subjects were instructed to use the right leg to exert a force of 63 N in the horizontal plane in twelve different directions, using a visual feedback on magnitude and direction. The force was measured using a force platform connected to the foot of the subject with a Nordic ski boot. Using this value of the force, the resultant moments at the joints were calculated in Reference [10] using a free-body diagram of the leg for each push direction.

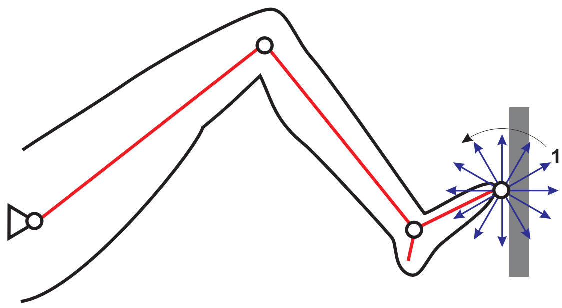

Figure 1. Human leg problem.

Surface EMG activity of the major leg muscles was recorded, full-wave rectified, low-pass filtered at 6 Hz, averaged over 1 sec and expressed as a fraction of the maximum EMG recorded in maximal isometric contractions at the same position. The physiological cross section area (PCSA) and the muscle moment arm nominal values are here derived from Reference [9].

For this problem, we consider a planar 3 DOF model of the human lower limb. The model has three frictionless revolute joints (hip, knee and ankle) and nine muscles: Tibialis Anterior (TA, ankle flexor), Soleus (SO, ankle extensor), Gastrocnemius (GA, ankle extensor, knee flexor), Vastii (VA, knee extensor), Rectus Femoris (RF, knee extensor, hip flexor), Short Head of Biceps Femoris (BFS, knee flexor), Long Head of Biceps Femoris (BFL, knee flexor, hip extensor), Iliacus (IL, hip flexor) and Gluteus Maximum (GLM, hip extensor). Muscles are modeled using the constitutive relation

$$f_i = K A_i a_i, \tag{19}$$

where f_i is the force in the i-th muscle, K represents the maximum muscle stress, whose nominal value is 40 N/cm^2, and A_i, a_i are i-th PCSA and activation, respectively, with $0 \leq a_i \leq 1$. Since this problem is a static one, more sophisticated muscle constitutive relations, describing the muscle force-length-velocity properties, are not required.

The optimization unknowns $\boldsymbol{x}$ are represented by the muscle activations

$$\boldsymbol{x} := (\ldots, a_i, \ldots)^T, \quad i = 1, \ldots, 9. \tag{20}$$

These are all bounded to remain within physical limits, i.e. $x_i \in [0,1], \forall i$. The experimental values $\boldsymbol{x}^{\mathcal{E}}$ correspond to the measured and post-processed EMG values. The PCSAs and the value of K are also taken as free optimization parameters, in the following indicated as $\boldsymbol{\pi}$, and are bounded as follows

$$A_i \in [0.6\bar{A}_i, 1.4\bar{A}_i], \tag{21}$$

$$K \in [0.5\bar{K}, 2.5\bar{K}], \tag{22}$$

where $\bar{A}_i$ indicates the nominal value of the PCSA for muscle i, and $\bar{K}$ is the nominal value of the maximum muscle stress. The equations of equilibrium for this problem can be written as

$$\boldsymbol{f}(\boldsymbol{x}; \boldsymbol{\pi}) = \boldsymbol{R}(\boldsymbol{\pi})\boldsymbol{x} - \boldsymbol{m} = 0, \tag{23}$$

where $\boldsymbol{m}$ are the known values of the moments at the joints, and $\boldsymbol{R}$ is a matrix that depends on the muscle moment arms and on parameters $\boldsymbol{\pi}$.

We consider a cost function search space that includes both muscle activations and muscle forces, and reads

$$J(\boldsymbol{x}; \boldsymbol{p}) := \sum_{k=2}^{N_k} p_{a,k} \sum_{i=1}^{9} x_i^k + \sum_{k=2}^{N_k} p_{f,k} \sum_{i=1}^{9} f_i^k(x_i, \boldsymbol{\pi}), \tag{24}$$

where f_i is given by the muscle constitutive equation (19). This expression of the search space is equipped with the normalization and non-negativity conditions

$$\sum_{k=2}^{N_k} (p_{a,k} + p_{f,k}) = 1, \tag{25}$$

$$p_{a,k} \geq 0, \quad k = 2, \ldots, N_k, \tag{26}$$

$$p_{f,k} \geq 0, \quad k = 2, \ldots, N_k. \tag{27}$$

The cost function parameters $\boldsymbol{p}$ are defined in this case as $\boldsymbol{p} := (\ldots, p_{a,k}, \ldots, p_{f,k}, \ldots)^T$, $k = 2, \ldots, N_k$.

To validate the implementation of the proposed procedures, we first impose a specific cost function and we compute the resulting muscle activations. The problem is solved for the nominal values of the modeling parameters, $\bar{\boldsymbol{\pi}}$, and it is formulated as

$$\begin{aligned} &\min_{\boldsymbol{x}} \sum_{k=2}^{N_k} p_{a,k} \sum_{i=1}^{9} x_i^k + \sum_{k=2}^{N_k} p_{f,k} \sum_{i=1}^{9} f_i^k(x_i, \bar{\boldsymbol{\pi}}) \\ &\text{s.t.: } \boldsymbol{R}(\bar{\boldsymbol{\pi}})\boldsymbol{x} - \boldsymbol{m} = 0, \\ &\qquad x_i \in [0,1], \quad i = 1, \ldots 9, \end{aligned} \tag{28}$$

for $N_k = 4$. In order to uniquely specify a cost function, we arbitrarily select

$$p_{a,2} = 1/2, \quad p_{a,3} = 0, \quad p_{a,4} = 1/2, \tag{29}$$

and

$$p_{f,k} = 0, \quad k = 2, 4. \tag{30}$$

The solution to problem (28) is assumed to represent the “experimental” values $\boldsymbol{x}^{\mathcal{E}}$. Given $\boldsymbol{x}^{\mathcal{E}}$, we now solve the cost function identification problem to see whether we identify the values of the unknown cost function parameters $\boldsymbol{p}$ that match the ones given in (29,30). To make the validation problem closer to a real identification problem, the initial guesses for the modeling parameters $\boldsymbol{\pi}$ were set to some random values, different from the nominal ones $\bar{\boldsymbol{\pi}}$.

The solution in terms of participation weights $p_{a,k}$ and $p_{f,k}$ is given in Figure 2 at top, which shows that the cost function identification algorithm recovers the same values that were arbitrarily assumed for solving the direct problem, exactly identifying the cost function. The bottom part of Figure 2 gives the muscle activations for the various push directions, showing that the reconstructed values, plotted using solid lines, exactly match the given ones, $\boldsymbol{x}^{\mathcal{E}}$, plotted using dashed lines. The modeling parameters $\boldsymbol{\pi}$ at convergence of the identification problem also exactly matched the nominal values used for the direct problem (28). In other words, the complete solution in terms of cost function, muscle activations and modeling parameters is exactly recovered by the proposed procedure, validating its implementation.

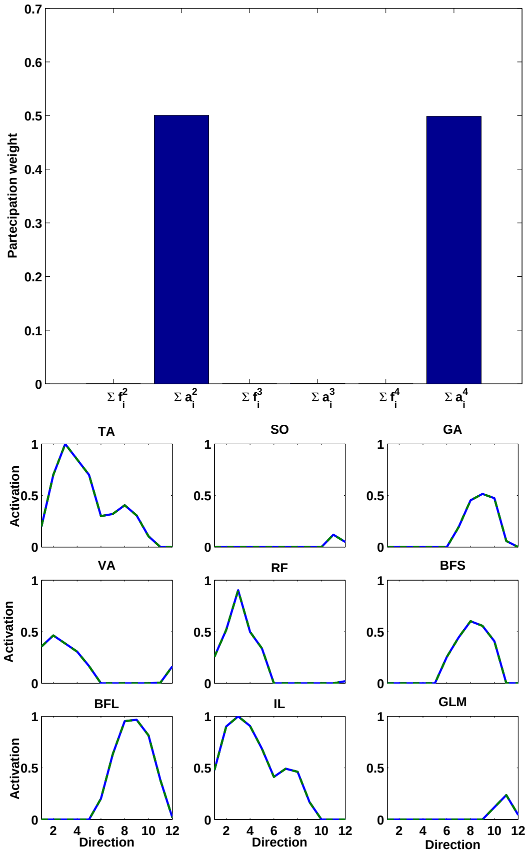

Figure 2. Top: participation weights $p_{a,k}$ and $p_{f,k}$ for the validation problem. Bottom: muscle activations a_i for the validation problem as functions of the push direction (solid line: computed activations; dashed line: $\boldsymbol{x}^{\mathcal{E}}$).

Next, we consider the same cost function identification problem with concurrent model tuning, but where the driving values $\boldsymbol{x}^{\mathcal{E}}$ are now the ones measured in the actual experiments of Reference [10]. Furthermore, we enlarge the search space by setting $N_k = 7$. The top part of Figure 3 gives the computed participation weights $p_{a,k}$ and $p_{f,k}$. Figure 3 at bottom gives the muscle activations for the various push directions. There is a reasonable match between the experimental (dashed lines) and computed (solid lines) activations, except for Soleus and Gluteus Maximum. The error between the computed and experimental activations is measured by the outer cost function, $||\boldsymbol{x} - \boldsymbol{x}^{\mathcal{E}}||/||\boldsymbol{x}^{\mathcal{E}}||$, whose value at convergence was equal to 0.2596 in this case.

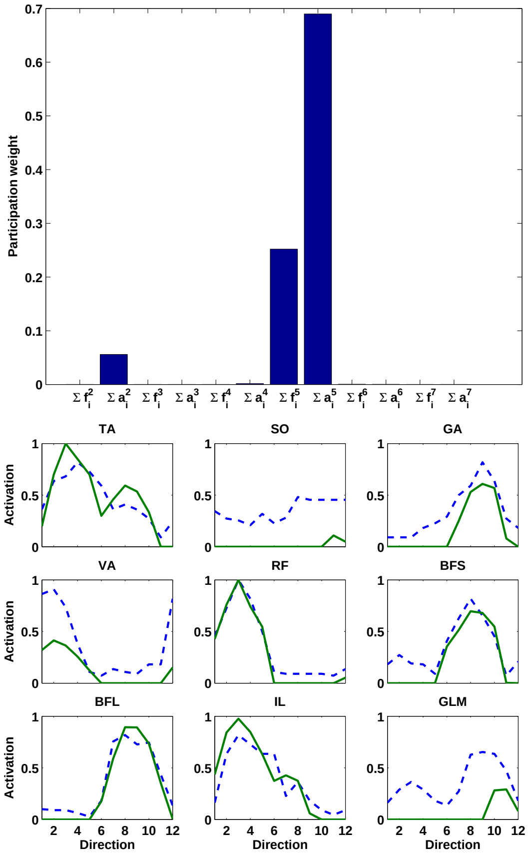

Figure 3. Top: participation weights $p_{a,k}$ and $p_{f,k}$ for the cost function search space of Equation (24). Bottom: muscle activations a_i as functions of the push direction for the cost function search space of Equation (24) (solid line: computed activations; dashed line: $\boldsymbol{x}^{\mathcal{E}}$).

To further explore possible forms of the search space, we consider next the expression

$$J(\boldsymbol{x};\boldsymbol{p}) := \sum_{k=2}^{N_k} p_{a,k} \sum_{i=1}^{9} x_i^k, \tag{31}$$

with its associated normalization and non-negativity conditions and where we set $N_k = 10$. This cost function accounts only for the activations, and not for the forces. The outer cost function value at convergence is 0.2611, and hence slightly larger than in the previous case. The participation weights $p_{a,k}$ are shown in Figure 4, top. The muscle activations are shown in Figure 4 at bottom.

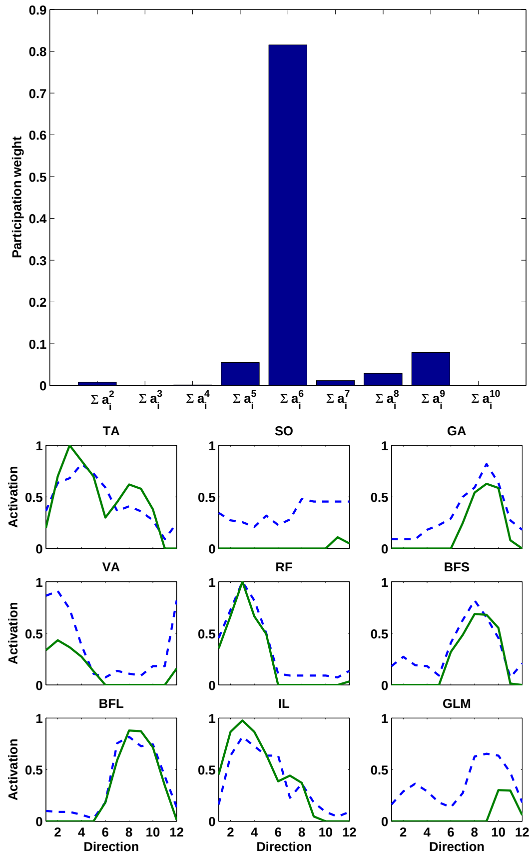

Figure 4. Top: participation weights $p_{a,k}$ for the cost function search space of Equation (31). Bottom: muscle activations a_i as functions of the push direction for the cost function search space of Equation (31) (solid line: computed activations; dashed line: $\boldsymbol{x}^{\mathcal{E}}$).

We consider now the expression

$$J(\boldsymbol{x};\boldsymbol{p}) := \sum_{k=2}^{N_k} p_{f,k} \sum_{i=1}^{9} f_i^k, \tag{32}$$

with the usual normalization and non-negativity conditions and where we set again $N_k = 10$. Hence, here we consider only the forces, and not the activations. The participation weights $p_{f,k}$ are shown at top in Figure 5, while the muscle activations are shown in the bottom part of the same figure. The participation weights show the predominance of the cubic term, as already noticed by many authors [1,5,8]. However, it is interesting to notice how in this case the solution does not follow too closely the experimental EMGs. This fact is confirmed by the value of the external cost function at convergence, which is equal to 0.3779, and hence substantially larger than in the previous cases.

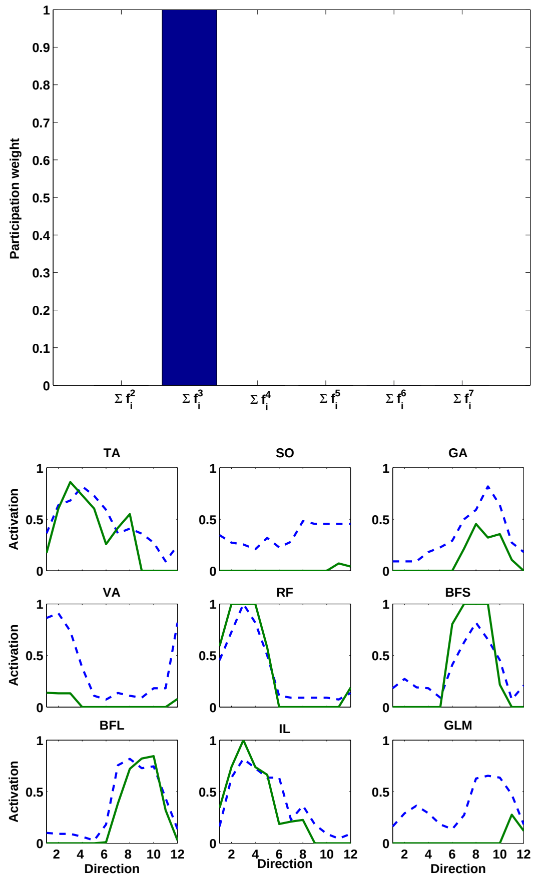

Figure 5. Top: participation weights $p_{f,k}$ for the cost function search space of Equation (32). Bottom: muscle activations a_i as functions of the push direction for the cost function search space of Equation (32) (solid line: computed activations; dashed line: $\boldsymbol{x}^{\mathcal{E}}$).

It appears that the best results are obtained when a combination of activations and forces is used. This might seem to indicate that multiple objective functions could be simultaneously optimized during this motor task. It is interesting to observe that the dominance of the cubic term in the case of sole muscle forces (Figure 5) disappears completely when more general cost functions are considered (Figure 3). Cubic force cost functions are often mentioned in the literature as providing good correlation with the experimental data (for review, see [8]); yet the results here presented show that the external cost, which measures the similarity between computed and experimental values, is substantially higher for the cubic force cost than in the case of the combined force-activation cost. While these results are certainly not sufficient to draw final conclusions, this observation seems to support the idea that the classical approach of hypothesizing up-front a cost function might lead to misleading conclusions regarding the principles governing motor control processes, and certainly deserves further investigation.

The lack of correspondence between the computed and experimental values for two of the muscles of the model should also be further investigated, as it might indicate a possible experimental or modeling problem. It is clear that this mismatch, even if confined to two single muscles, could impact the identification of the cost function. Probably some of these issues could be solved using experimental measures of higher precision. It is in fact clear that the whole identification procedure is driven by the experimental data $\boldsymbol{x}^{\mathcal{E}}$; if this data is not of sufficient accuracy, even in the presence of a reasonably accurate mathematical model of the bio-system, the final outcome of the computation will be affected.

CONCLUSIONS

In this work we have proposed a methodology for inferring the cost functions that underlie experimentally observed motor tasks. The approach is based on mathematical models of both the bio-system and of the possible cost functions that were presumably used to accomplish a given task. On the basis of experimental data, the cost function that best approximates the observations with the given models is computed.

The method described here can be used for both dynamic and static problems in biomechanics. We have shown that in both cases the numerical procedure leads to the solution of a nested parameter optimization problem. To make the problem tractable, we have proposed to explicitly express the necessary optimality conditions of the inner optimization problem, transforming the overall cost function identification into a classical NLP problem, which can then be solved using standard available techniques.

We have preliminary tested the new method on a representative case, that seems to indicate its potential applicability to more complex problems. In principle, we think that this method might provide new insight into the organization of motor control, by providing a way to measure otherwise hidden important characteristics of the neuro-musculoskeletal system.

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