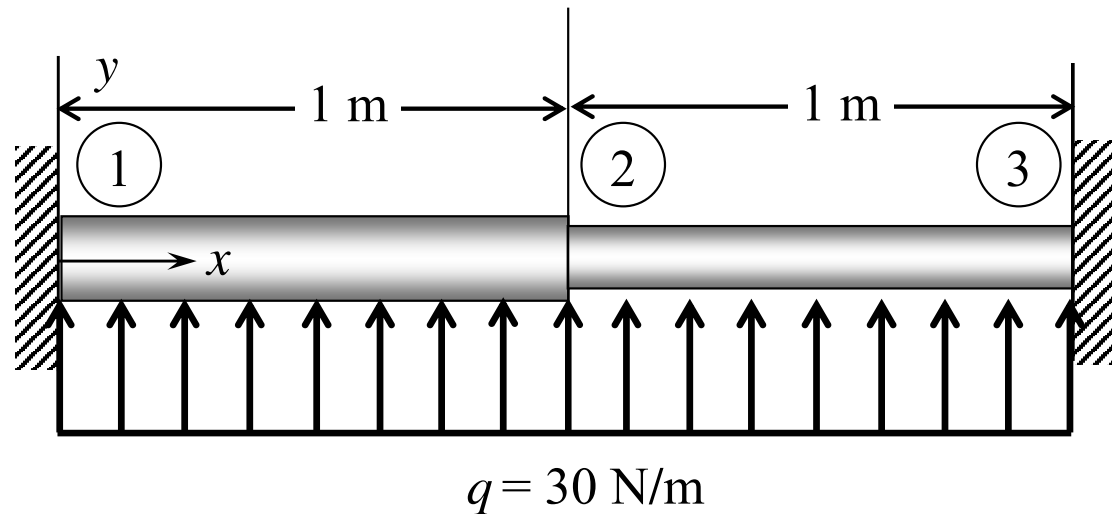


**Tutorial 3:**

**Plane Beam**

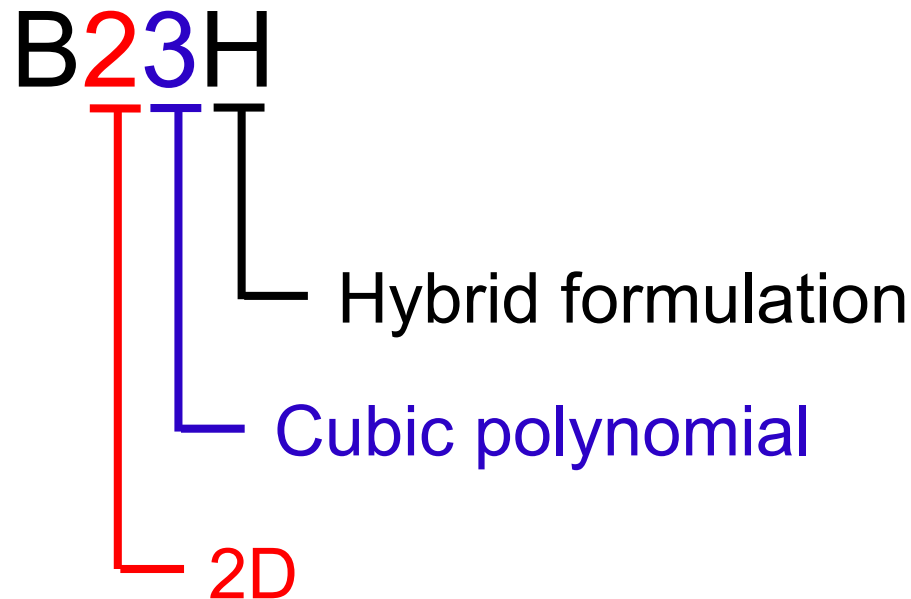
# CLAMPED-CLAMPED BEAM

- $E = 100 \text{ MPa}$ ,
- Circular section with  $r_1 = 20\text{mm}$ ,  $r_2 = 15\text{mm}$
- Plot bending moment and shear force diagrams



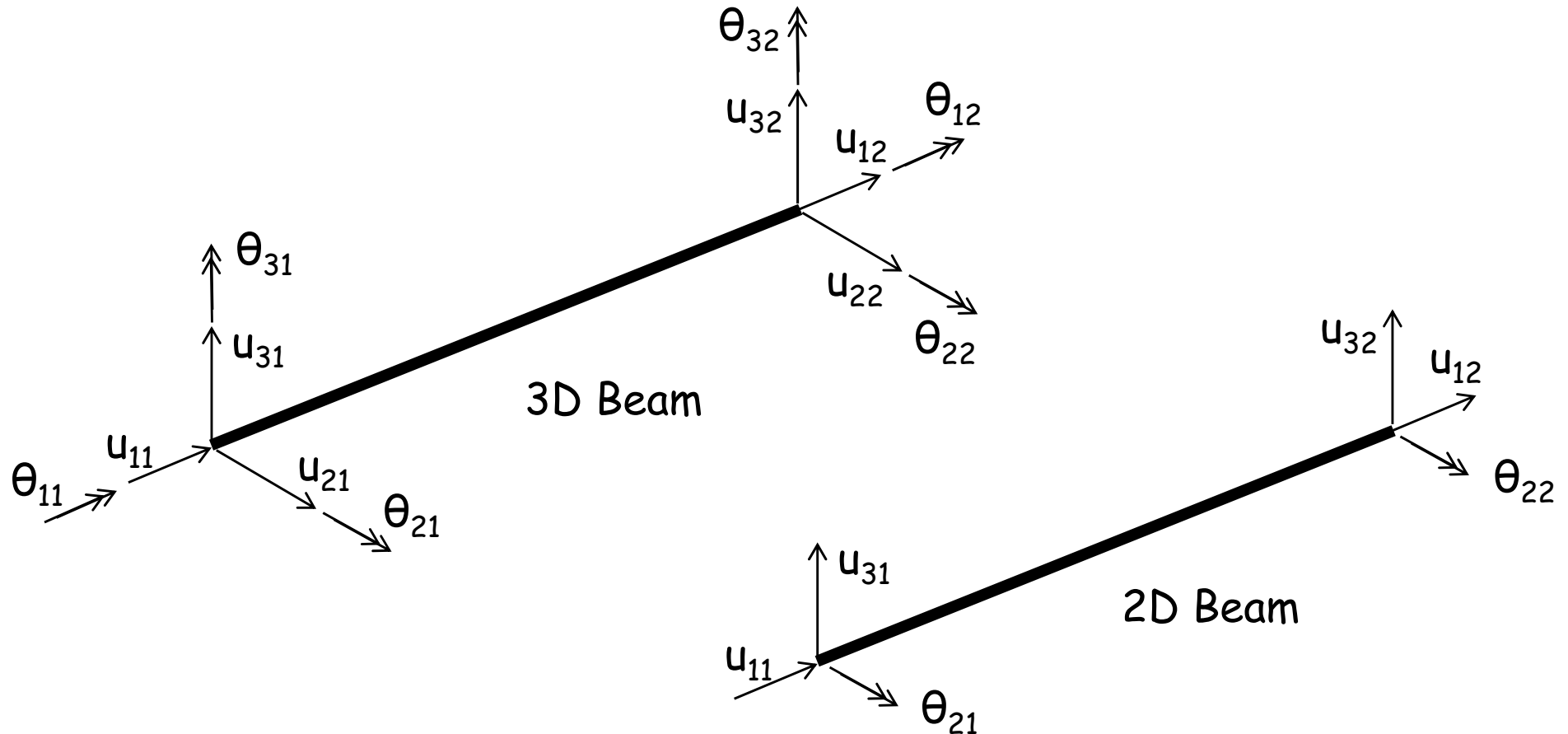
# ABAQUS BEAM ELEMENT

- Classification of Beam Elements
  - Dimension (2D / 3D)
  - Formulation type (Mindlin-2node / Mindlin-3node / Euler)  
Mindlin (Shear flexible), Euler (Cubic formulation)
  - Additional information (OS (open section) / H (hybrid) / OSH)



# 2D VS 3D BEAM ELEMENTS

- Degree of freedoms for a beam element
  - 2D/3D (2D beam is a special case of the 3D beam)
  - With translation (from the truss element)
  - Do not confuse sign convention for displacements and forces



# GEOMETRY MODELING

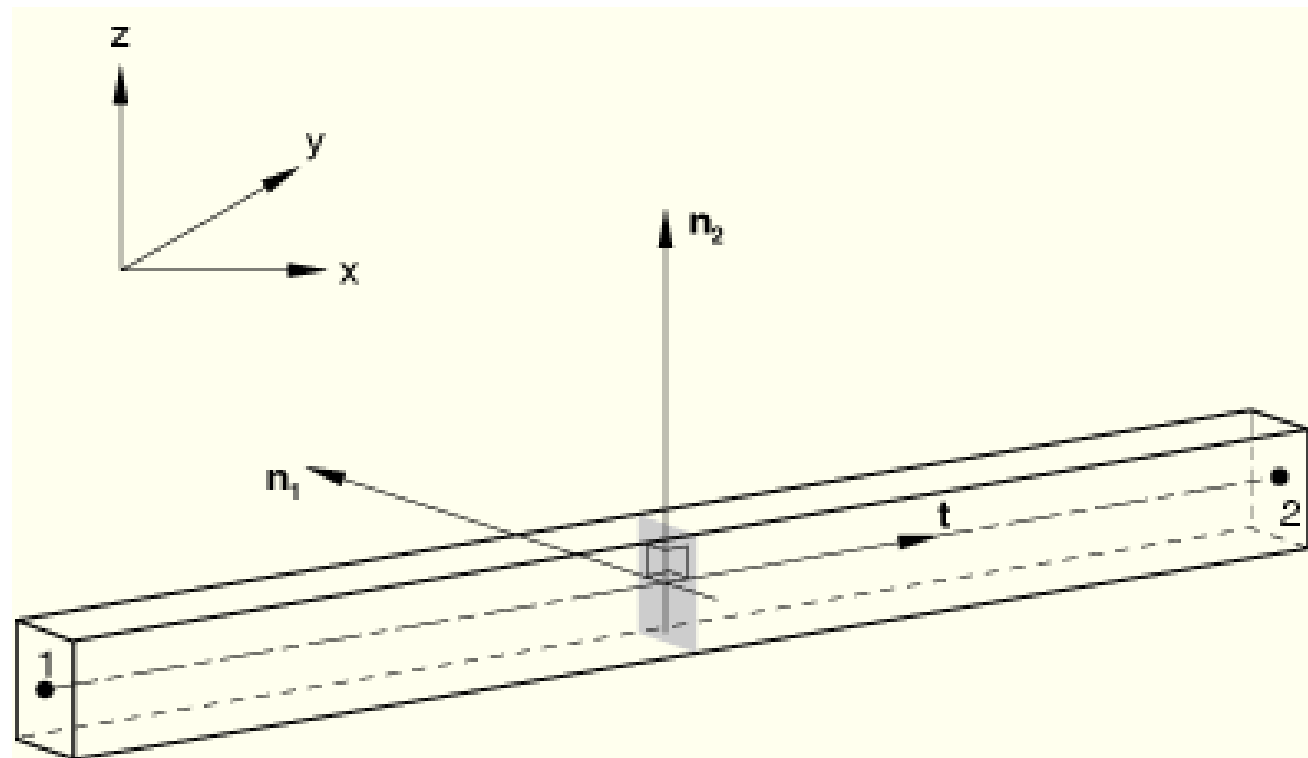
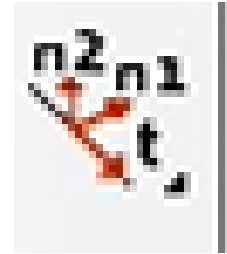
- Parts
  - 2D Planar, Deformable, Wire, App Size = 4
  - Create lines: (-1, 0), (0,0), (1, 0)
- Materials
  - Mechanical, Elasticity, Elastic
  - Young's modulus = 100E6, Poisson's ratio = 0.3
- Profiles
  - Name: R20, Circular, Continue,  $r = 0.02$ , Ok
  - Name: R15, Circular, Continue,  $r = 0.015$ , Ok
- Sections
  - Name: R20, Beam, Beam, Continue, Profile name: R20, Ok
  - Name: R15, Beam, Beam, Continue, Profile name: R15, Ok

- Section Assignment

- Part, Part-1, Section Assignments
- Select left beam, Done, Section: R20, Ok
- Section Assignments, Select right beam, Done, Section R15, Ok

# SECTION ORIENTATION

- Beam cross section needs orientation (ex. Second moment of inertia)
- $n_1$  vector is  $(0, 0, -1)$  for plane beam (may not be modified for plane beam)
- Tangent direction vector  $\mathbf{t}$  is dependent to the direction of wire geometry



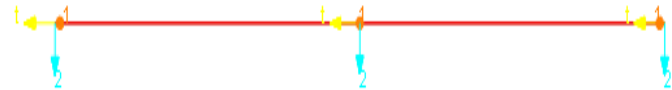
# SECTION ORIENTATION cont.

## CASE 1

Direction of wire geometry (t)



Assigned beam orientation

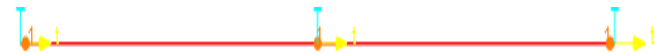


## CASE 2

Direction of wire geometry (t)



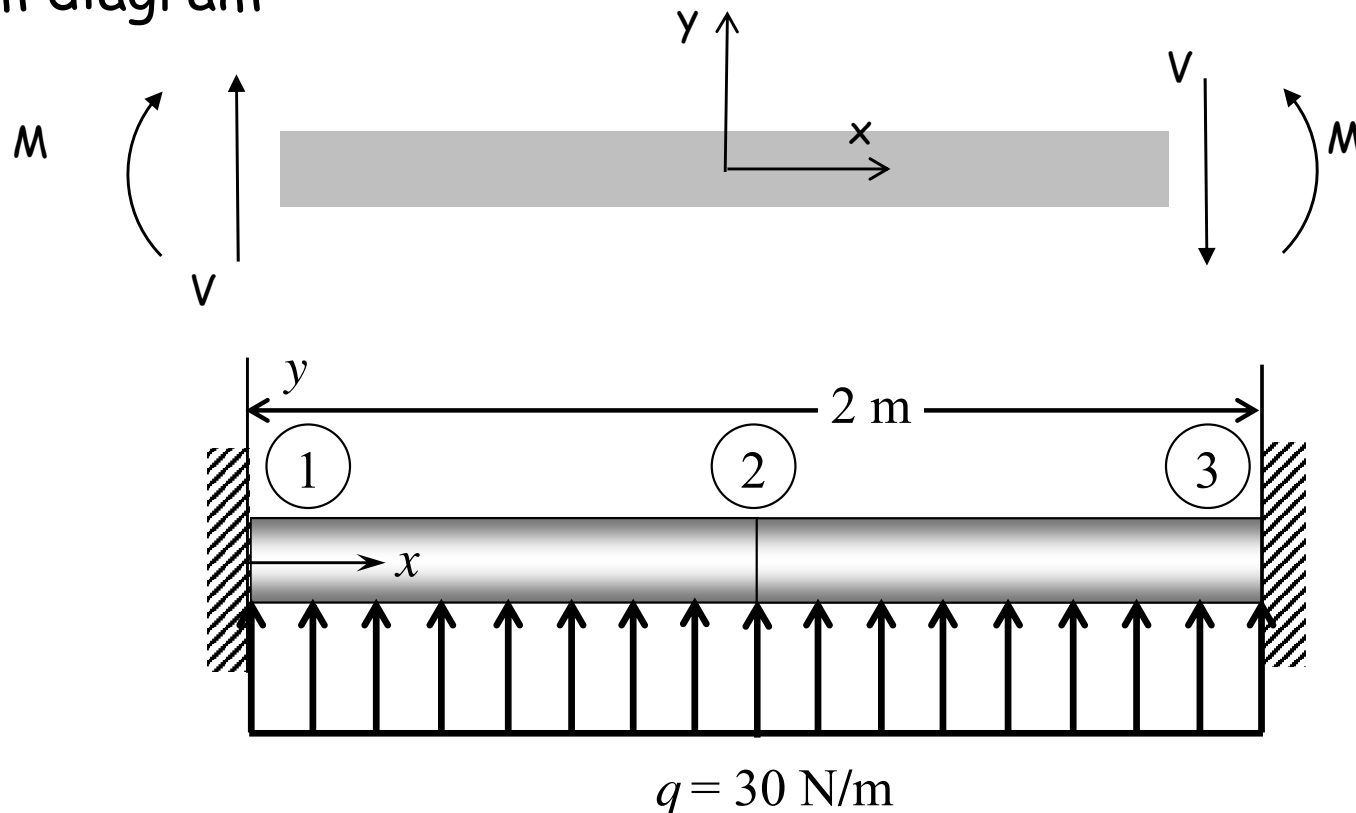
Assigned beam orientation





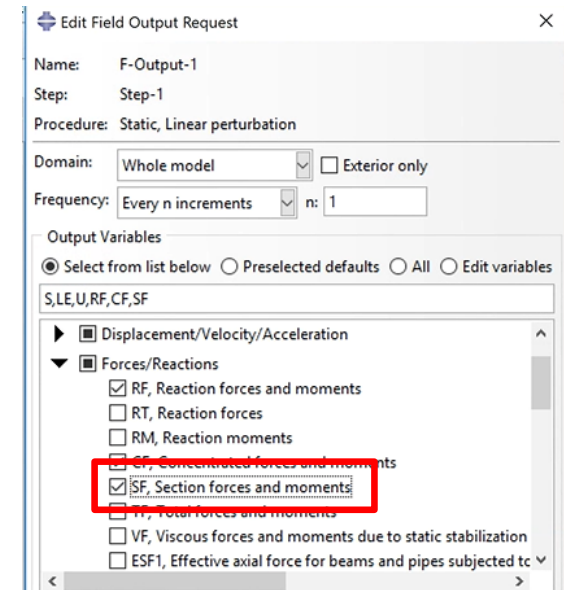
# SIGN CONVENTION

- Sign Convention for force and moment
  - Must not be confused with sign convention for displacements
  - Users must make sure which sign convention is used in a FEA software
  - Shear force and bending moment sign convention of ABAQUS for beam diagram



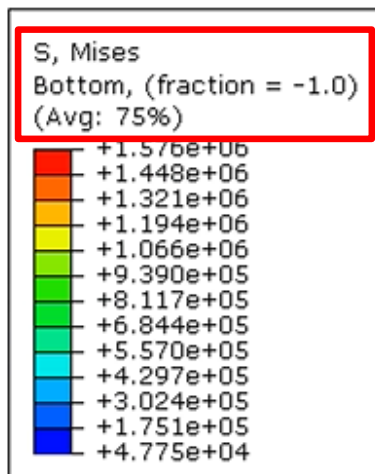
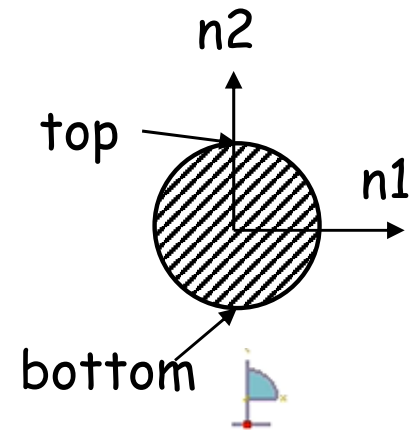
# SIMULATION SETUP

- Assembly, Instance
- Steps
  - Linear perturbation, Static
- Field output request
  - Check SF, Section forces and moments
- BCs
  - Initial, Encastre (clamped)
- Loads
  - Mechanical, Pressure (force/length for beam) or Line load (force/length), select upward, Uniform, 30
- Mesh
  - Element type, "Classical beam theory" or Euler beam theory, Cubic polynomial, (B23), Global element size = 0.2.
  - Generate elements: Mesh/Parts



# POSTPROCESSING

- Analysis, Create Job, Data Check, Submit
- Results
- Deformed plot, Stress plots
  - Result/Field output, Section points, Top or Bottom
  - View/ODB Displace Option/Render beam profiles

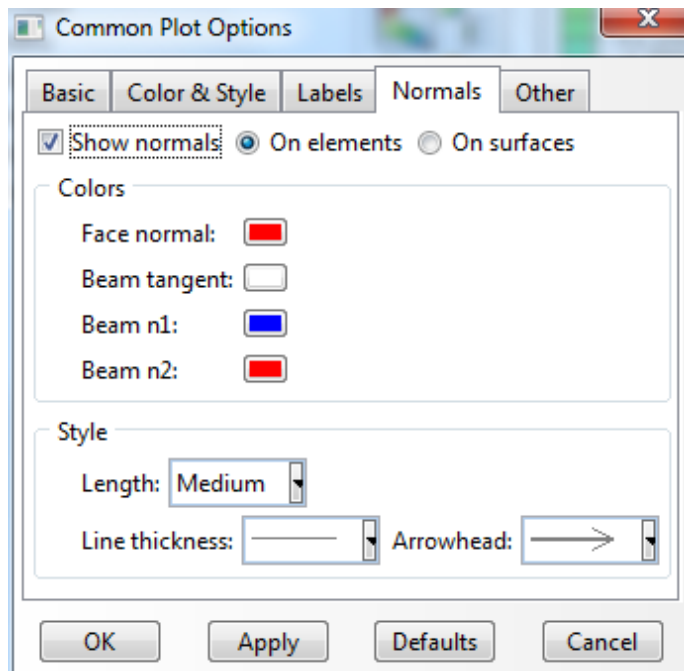


ODB: Job-1.odb Abaqus/Standard 3DEXPERIENCE R2019x Tue Oct 08 11:05:49 Eastern Daylight Time 2019  
Step: Step-1  
Increment 1: Step Time = 2.2200E-16  
Primary Var: S, Mises

# SHOWING ELEMENT NORMAL

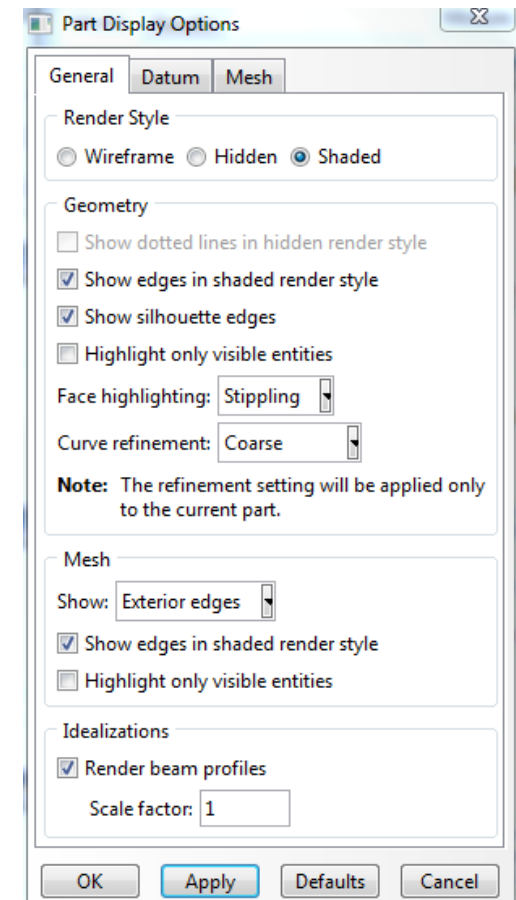
- Common Plot Options

- Normals, check "Show normals", "On element"



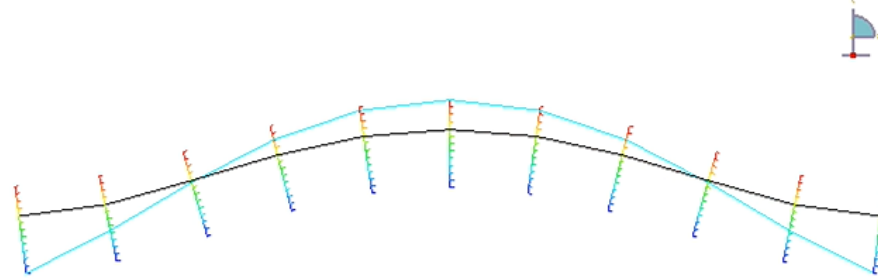
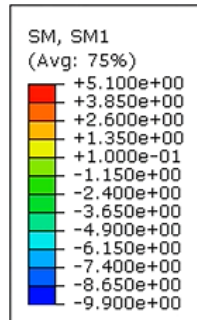
- Module part, View, Part Display Options

- check "Render Beam Profile"



# BENDING MOMENT DIAGRAM

- Result/Field Output/SM
- Contour Plot Options/Show tick marks for line elements

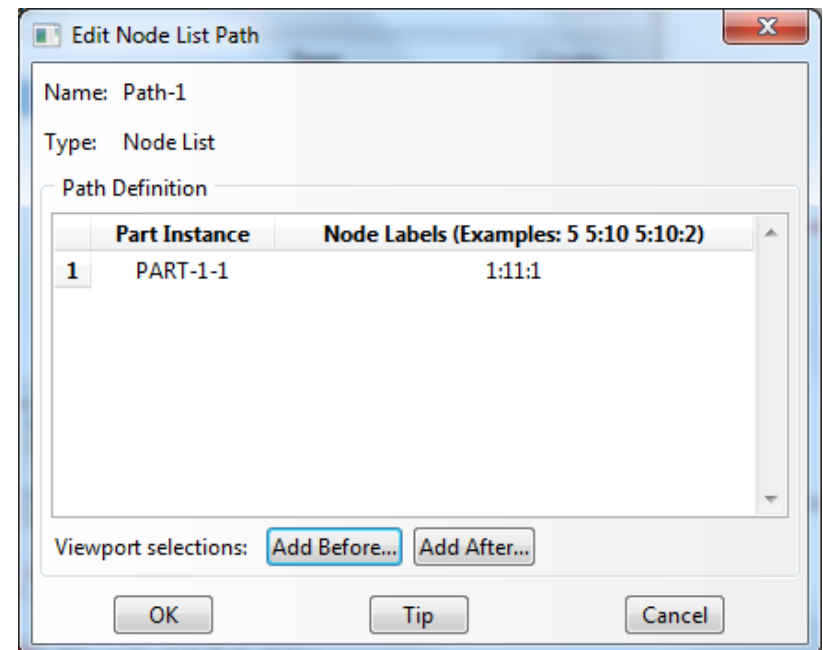


Bending  
moment  
diagram



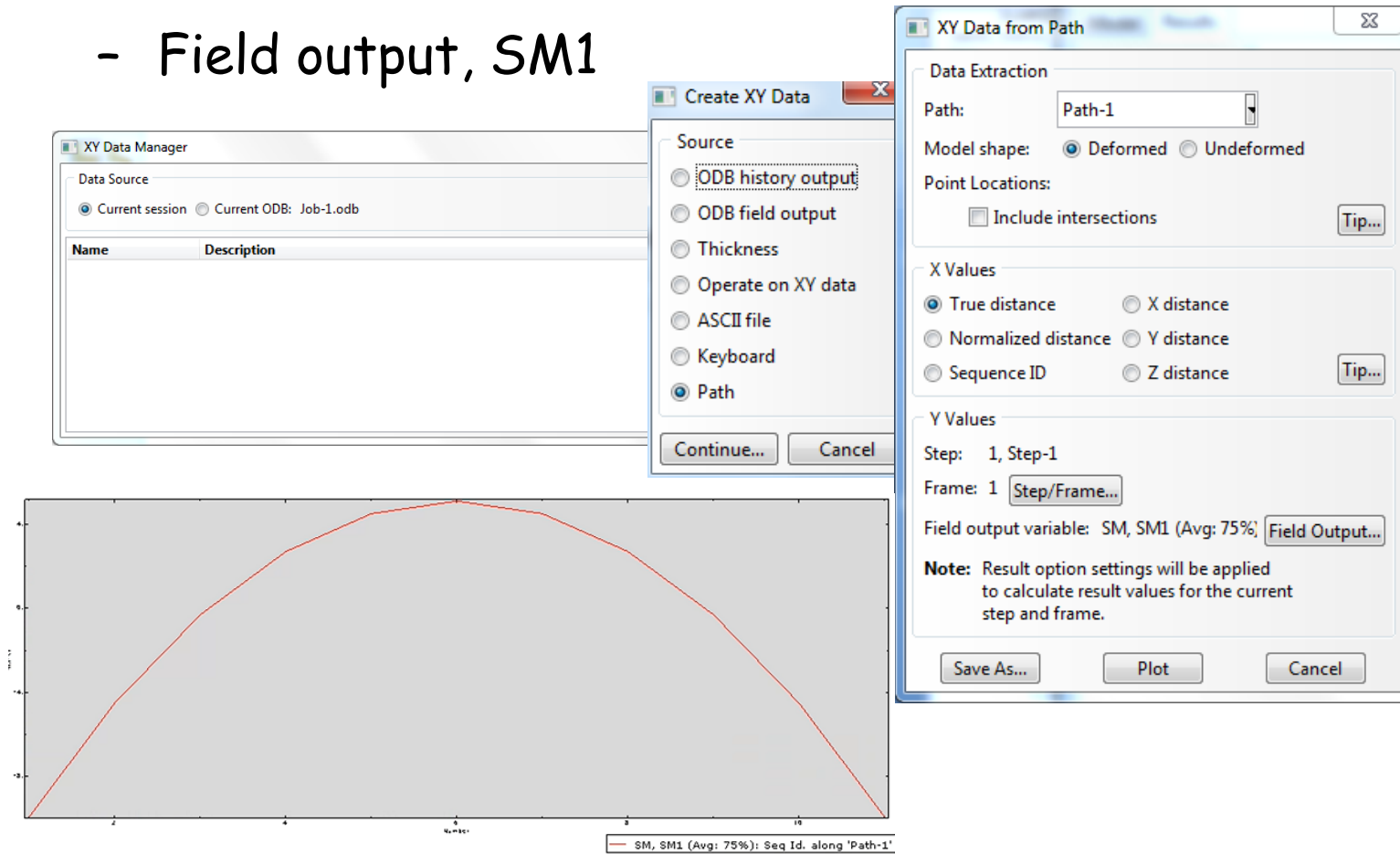
ODB: Job-1.odb Abaqus/Standard 3DEXPERIENCE R2019x Tue Oct 08 11:05:49 Eastern Daylight Time 2019  
Step: Step-1  
Increment 1: Step Time = 2.2200E-16  
Primary Var: SM, SM1

- Shear forces (SF2) are not available for B23 element
- Paths, Node list  
(first node #, last node #, inc)



# BENDING MOMENT DIAGRAM cont.

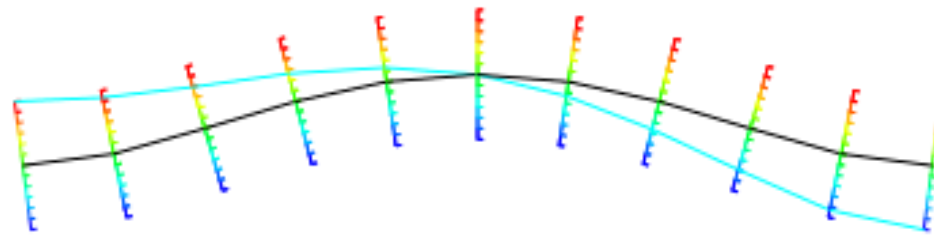
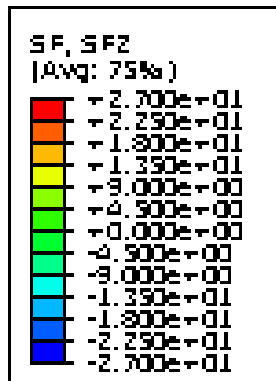
- XYData, path, X Values = Sequence ID
  - Field output, SM1



- To adjust font size of the XYData plot
  - Double click object to open a dialog box to adjust its properties (ex: double click the legend box to enlarge its font size)

# SHEAR FORCE IN TIMOSHENKO BEAM

- Change element type
  - Element type, "Timoshenko beam theory" or Mindlin beam theory, Shear flexible, (B21), Global element size = 0.2
- Field Output
  - Shear force output (SF2) is available

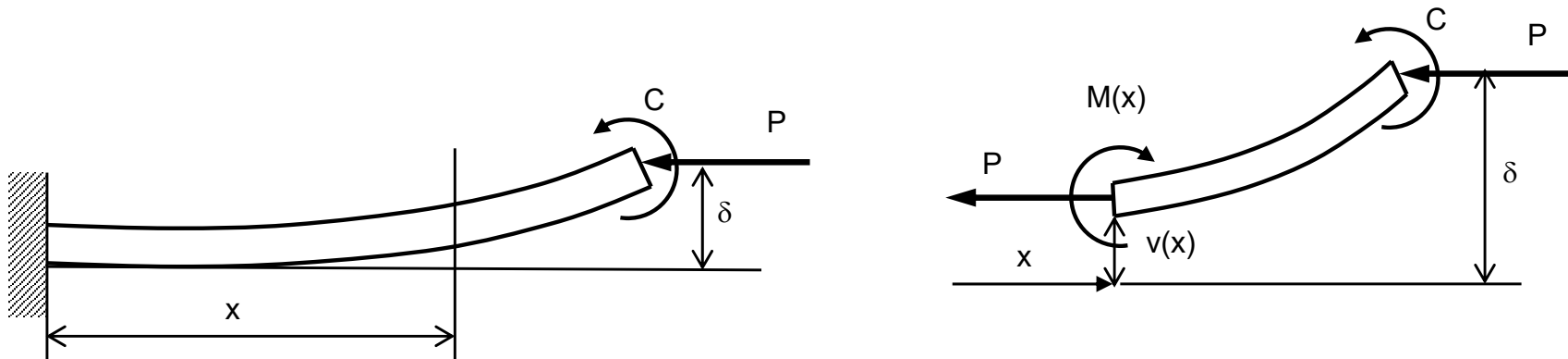


# Buckling Tutorial



# Buckling of Column

- In plane beam, we assume that axial deformation (bar) is independent of bending deformation (beam)
- This is normally true, but when the deformation is large or the force is large, the coupling effect become important



- Buckling: beam collapses under compressive axial load
- **Critical load**
  - When the compressive load is less than the critical load, the beam deforms as a bar
  - When the load reaches the critical load, the beam suddenly collapses

# Load Factor

- For a given applied load  $F$ , each element has axial element force  $P^{(e)}$

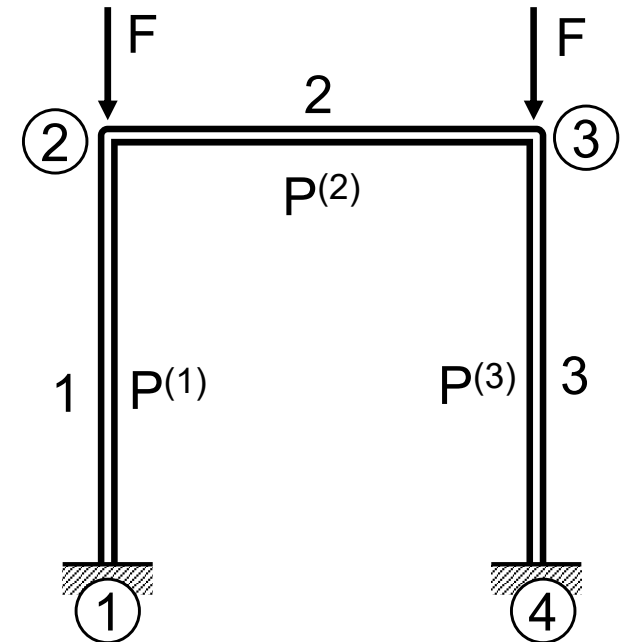
- Load Factor

- Let's assume that the structure is stable at the current load  $F$
- How much the load can increase before buckle?

$$P_{cr} = \lambda \cdot F \quad \rightarrow \text{Load factor}$$

- Due to linearity, element forces will also be  $\lambda \cdot P^{(e)}$
- All elements will have different  $P^{(e)}$ , but  $\lambda$  will be the same
- The goal is to find  $\lambda$  when the structure becomes unstable

Instability  $\Rightarrow$  Large (infinite) displacement



# Finite Element Equation for Buckling

- Stiffness + incremental stiffness

$$[\mathbf{K} - \lambda \mathbf{K}_{inc}] \{\mathbf{Q}\} = \{\mathbf{F}\}$$

$$[\mathbf{k}_{inc}^{(e)}] = \frac{P^{(e)}}{30L} \begin{bmatrix} 36 & 3L & -36 & 3L \\ 3L & 4L^2 & -3L & -L^2 \\ -36 & -3L & 36 & -3L \\ 3L & -L^2 & -3L & 4L^2 \end{bmatrix} \begin{matrix} v_i \\ \theta_i \\ v_j \\ \theta_j \end{matrix} \quad \text{Incremental stiffness}$$

- The stiffness is reduced due to coupling effect of compressive load

$$P_{cr} = \lambda \cdot F$$

$\lambda$  : Load factor, eigenvalue

$P^{(e)}$  : Element force

$P_{cr}$  : Critical load

# Finite Element Equation for Buckling

$$[\mathbf{K} - \lambda \mathbf{K}_{inc}] \{\mathbf{Q}\} = \{\mathbf{F}\}$$

- This matrix equation has a small displacement  $\{\mathbf{Q}\}$  if  $[\mathbf{K} - \lambda \mathbf{K}_{inc}]$  is positive definite
- Instability occurs when the matrix is singular

$$|\mathbf{K} - \lambda \mathbf{K}_{inc}| = 0$$

- Among many eigenvalues, the lowest one will be used to calculate the critical load

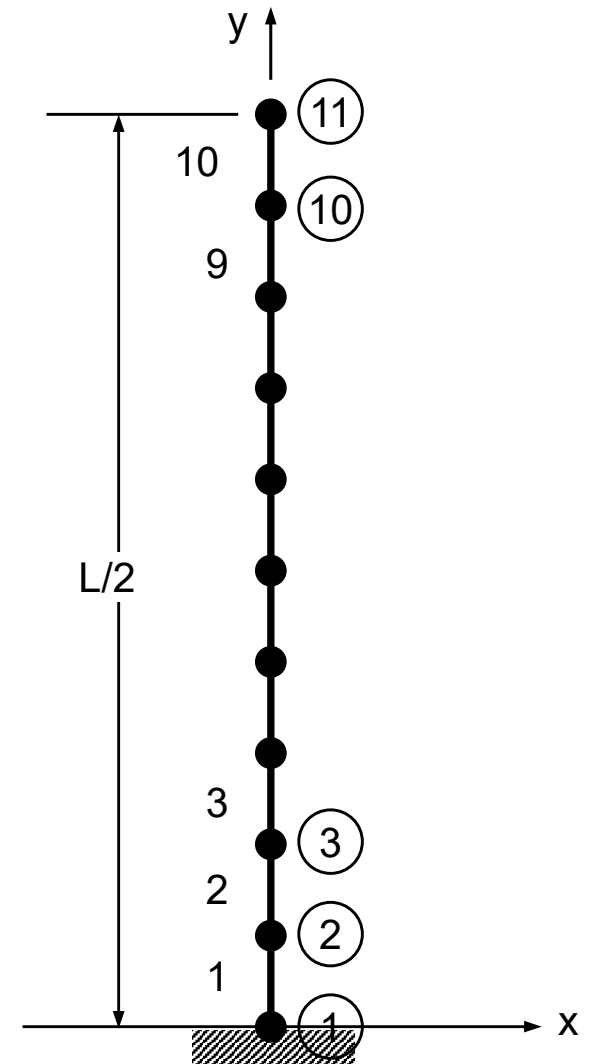
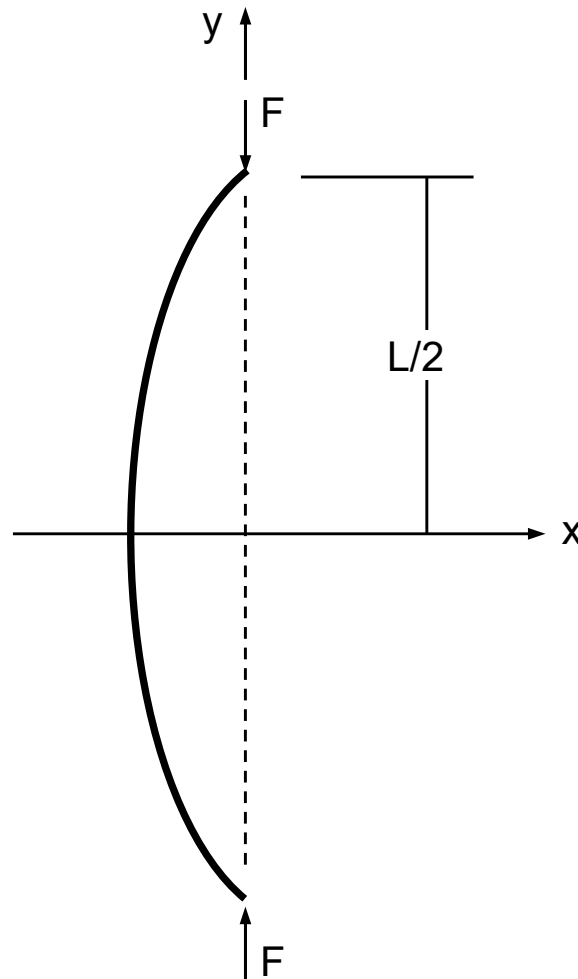
$$P_{cr} = \lambda \cdot F$$

# Buckling with Hinged Ends

- $E = 20E6$  psi,  $L = 200$ in,  $F = 1$  lb, square section 0.5"X0.5"
- Theoretical buckling load = 25.7 lb

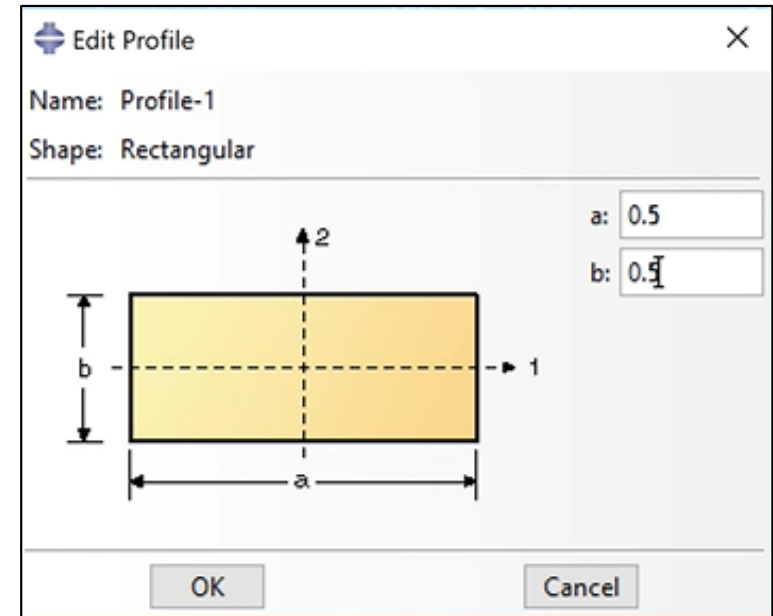
$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

- Use symmetry (half model)




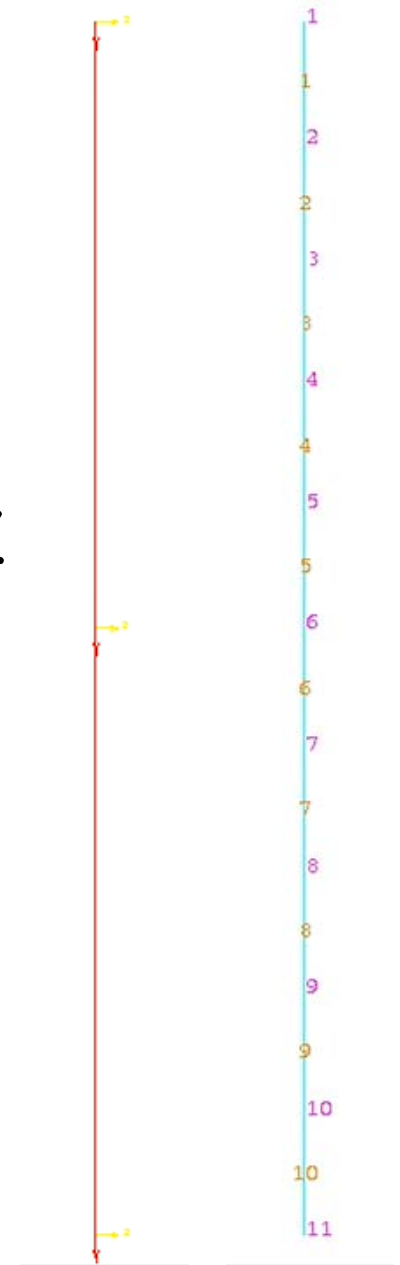
# GEOMETRY MODELING

- Parts, 2D Planar, Deformable, Wire, Approximate size = 200
- Create lines, (0,100), (0,0), Done
- Materials, Mechanical, Elasticity, Elastic, Young's modulus =  $2E7$ , Poisson's ratio=0.3, OK
- Profiles, Rectangular,  $a=0.5$ ,  $b=0.5$
- Sections, Beam,  
Profile name = Profile-1  
Material name = Material-1
- Part, Section Assignment,  
"Select the line", Done,  
Section = Section-1



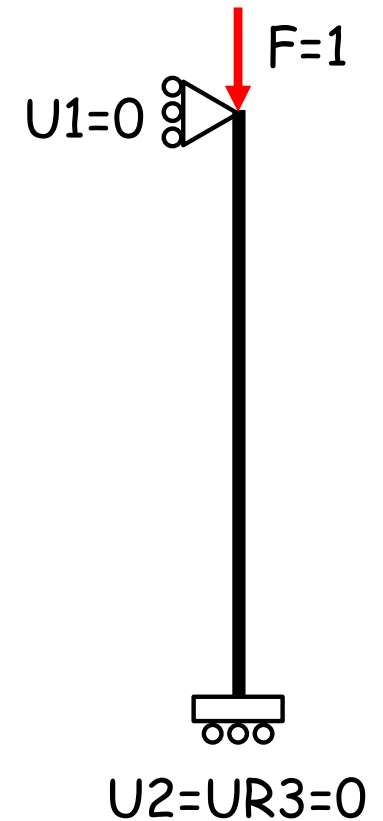
# MESHING

- Module = Property
- Assign beam orientation,  "Select the line", Done, n1 direction = (0, 0, -1), OK
- Parts, Mesh
- Mesh, Element Type, "Select the line", Done, Standard, Beam, Linear, Cubic formulation, OK
- Seed, Edges, "Select the line", Done, By number, Number of elements = 10, OK
- Mesh, Part..., Yes
- View, Part Display Option, Mesh, Show node labels, Show element labels, OK



# MESHING

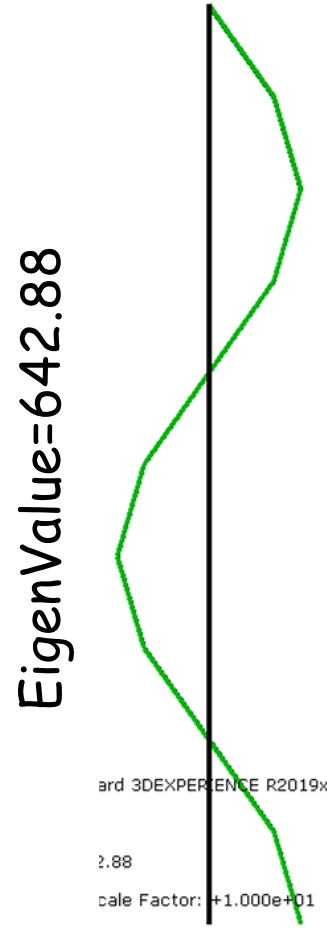
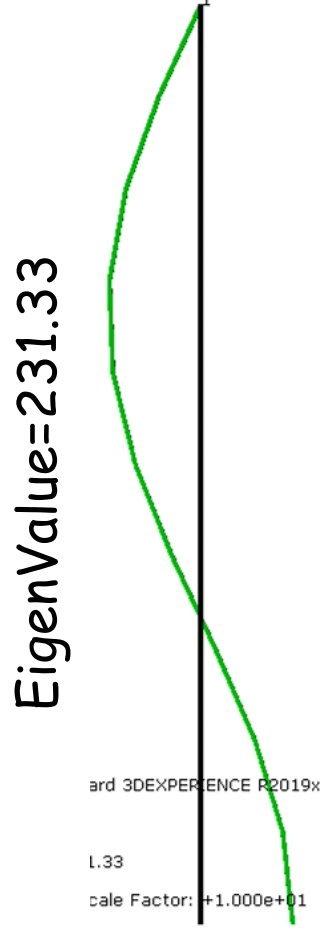
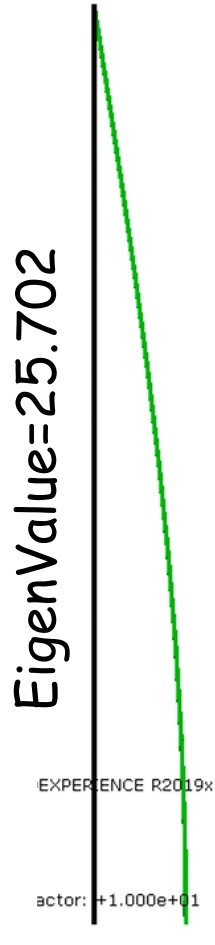
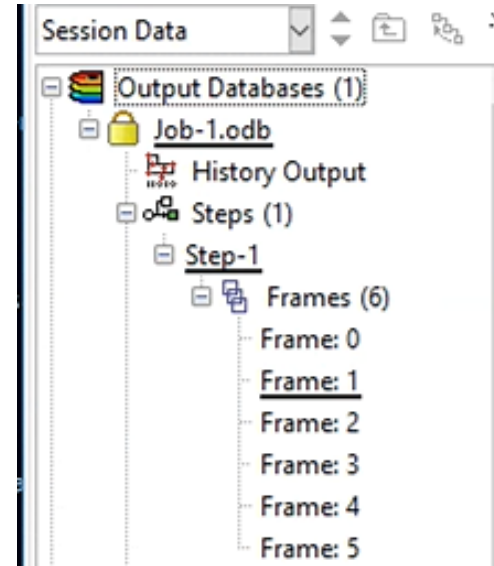
- Assembly, Instance, Part-1, OK
- Steps, Linear perturbation, Buckle, Number of eigenvalues requested = 5
- BCs, Step=Initial, Displacement/Rotation, "Select top node", Done, U1, OK
- BCs, Step=Initial, Displacement/Rotation, "Select bottom node", Done, U2, UR3, OK
- Loads, Step-1, Concentrated force, "Select top node", Done, CF2=-1
- Jobs, Job-1, Continue..., OK
- "Right click on Job-1", Submit
- "Right click on Job-1", Results





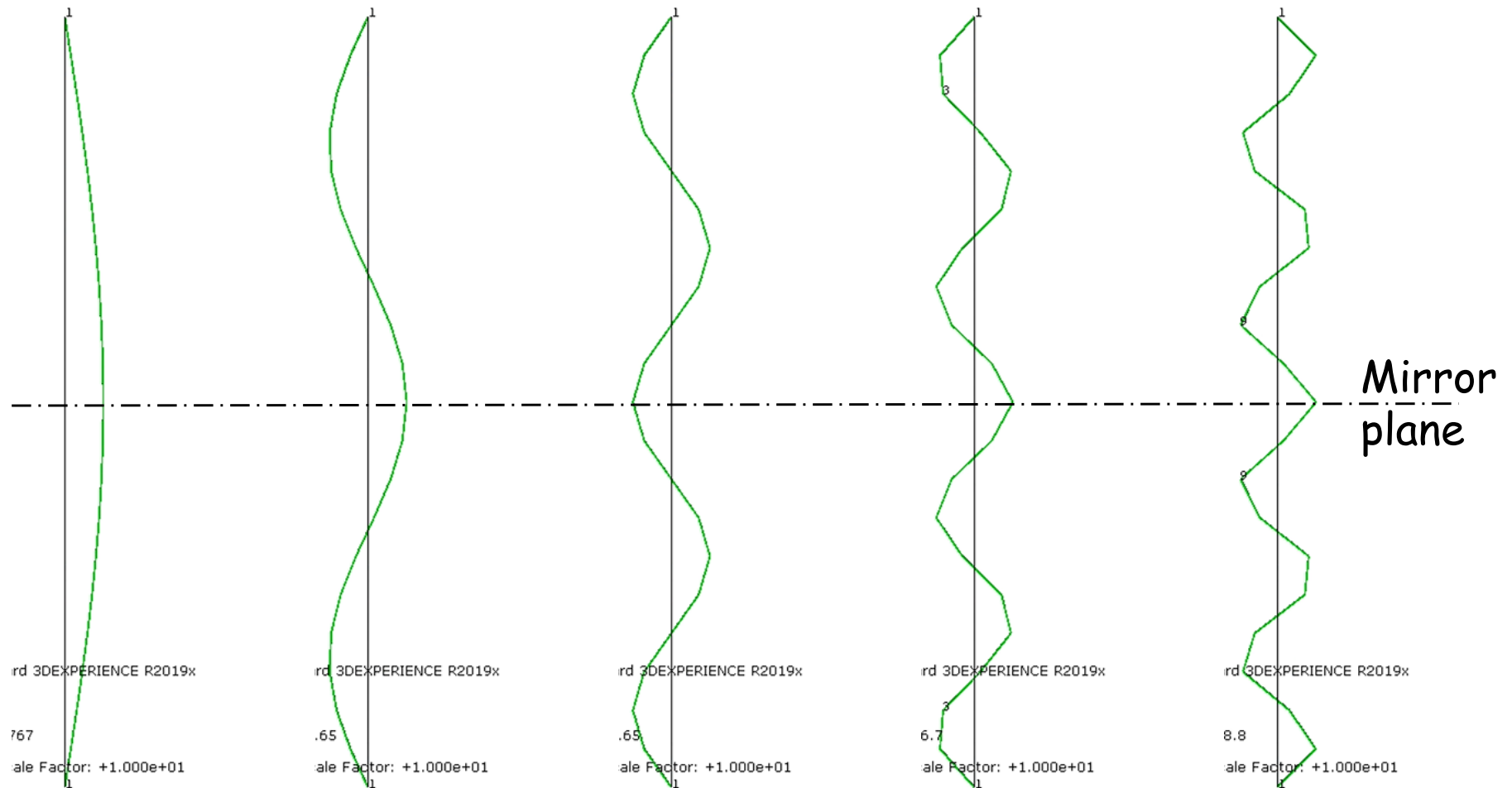
# POSTPROCESSING

- Output Database, Job-1.odb, Steps, Step-1, Frames, "Select Frame:1"
- Plot Deformed Shape, "EigenValue = 25.702"
- Repeat for other frames



# MIRROR IMAGE

- View, ODB Display Options, Mirror/Pattern, Mirror plane = XZ



# UNDERSTAND BUCKLING LOAD

Eigenvalue		Buckling load
F=1 lb	F=10 lb	$P_{cr}$
25.702	2.5702	25.702
231.33	23.13	231.33
642.88	64.28	642.88
1261.8	126.18	1261.8
2092.4	209.24	2092.4

Buckling Load = Applied force \* Eigenvalue

The lowest buckling load is the most important!!