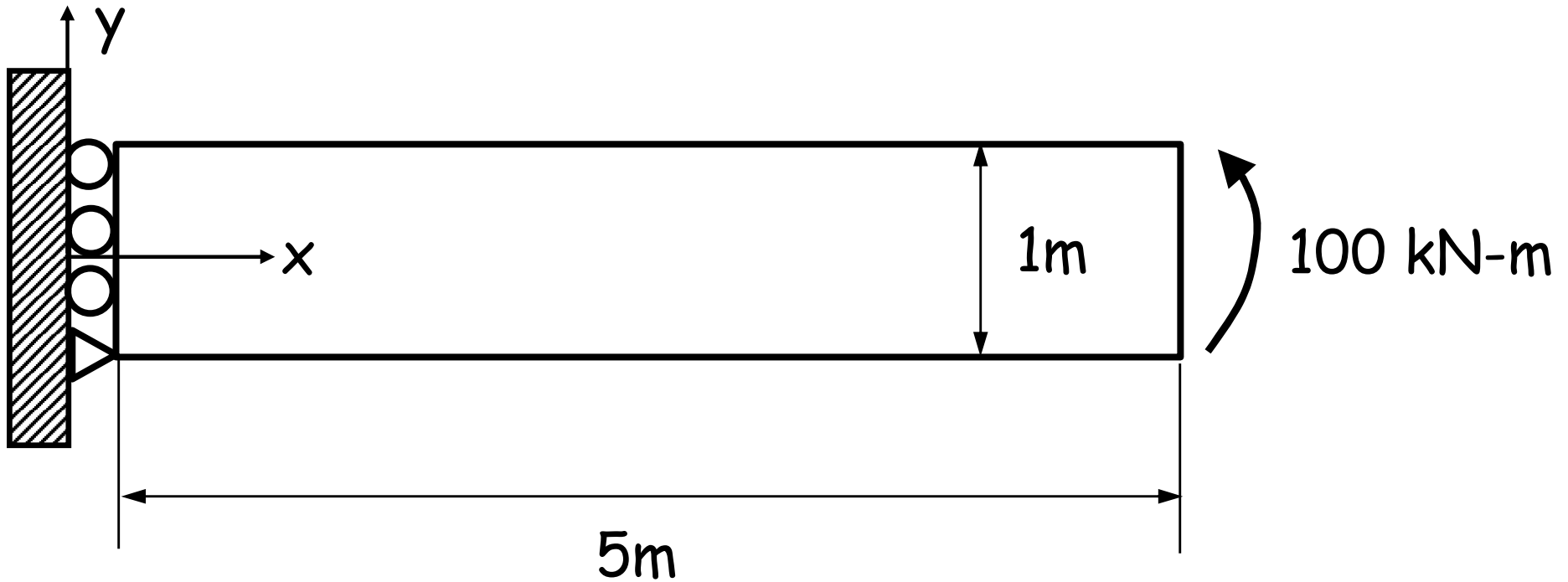


**Tutorial 4:**  
**2D Plane (CST/Q4)**

# CANTILEVER BEAM

- $E = 200 \text{ GPa}$ ,  $\nu = 0.3$
- Thickness  $t = 0.01 \text{ m}$



# Preliminary Analysis

- Pure bending
  - Bending moment is constant in  $x$
  - Stress is constant in  $x$
  - Linearly varying stress in  $y$

$$\sigma_{xx} = -\frac{My}{I} = -\frac{My}{\frac{bh^3}{12}} = -\frac{12 \times 10^5}{0.01} y$$

$$(\sigma_{xx})_{\max} = -\frac{12 \times 10^5}{0.01} (-0.5) = 60 \times 10^6 = 60 \text{MPa}$$

- Tip deflection

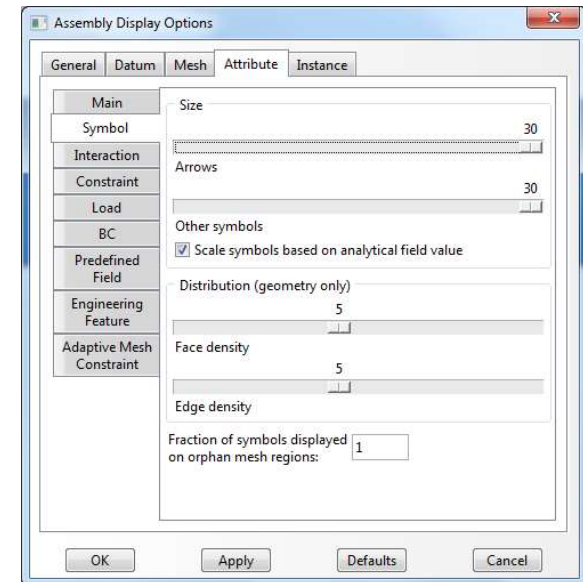
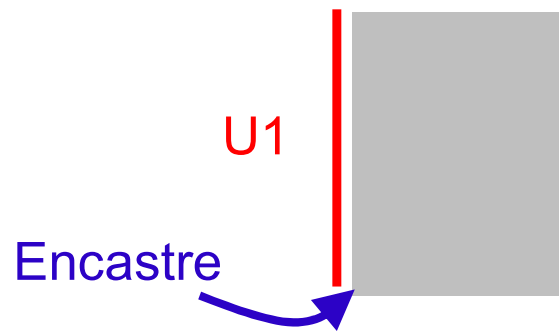
$$\delta_{\max} = \frac{ML^2}{2EI} = \frac{10^5 \times 5^2}{2 \times 2 \times 10^{11} \times \frac{0.01 \times 1^3}{12}} = 7.5 \times 10^{-3} = 7.5 \text{mm}$$

# CANTILEVER BEAM (CST)

- Parts
  - 2D Planar, Deformable, Shell, App Size = 10
  - Create lines (rectangle): (0, 0), (5, 1)
- Materials
  - Mechanical, Elasticity, Elastic
  - Young's modulus = 200E9, Poisson's ratio = 0.3
- Sections
  - Solid, Homogeneous
  - Set plane stress/strain thickness to 0.01 m
- Assign the section to the part

# CANTILEVER BEAM (CST)

- Assembly, Instance
- Steps
  - Linear perturbation, Static
- BCs
  - Initial, Encastre + Displacement/Rotation, U1

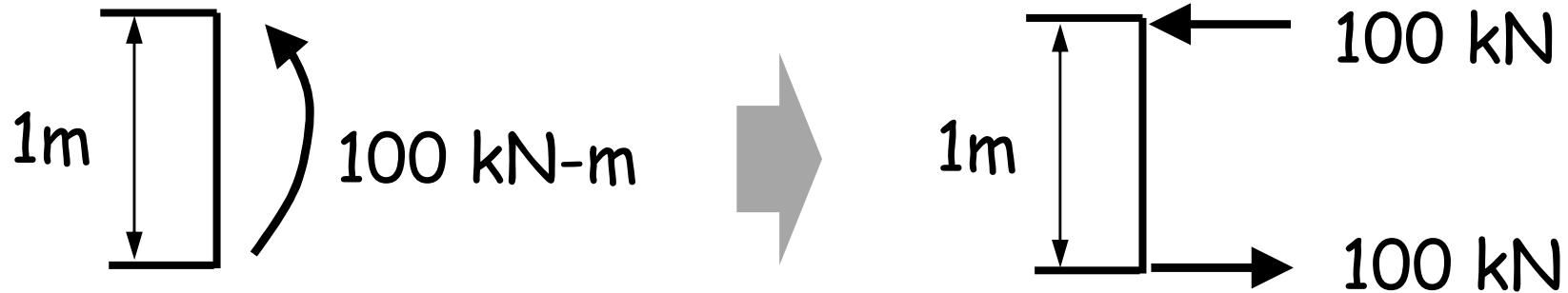


- Tip
  - To change BC symbols: View, Assembly Display Options, Attribute

# CANTILEVER BEAM (CST)

- Loads

- Mechanical, Pressure, select upward, Uniform, 30



- Mesh

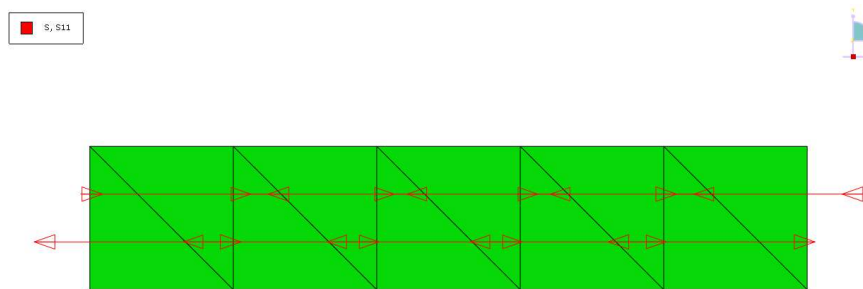
- Assign Mesh Controls, Tri (Tri only)
- Global element size = 1

- Mesh control

- Edit Mesh, Element, Swap diagonal

# CLAMPED-CLAMPED BEAM (CST)

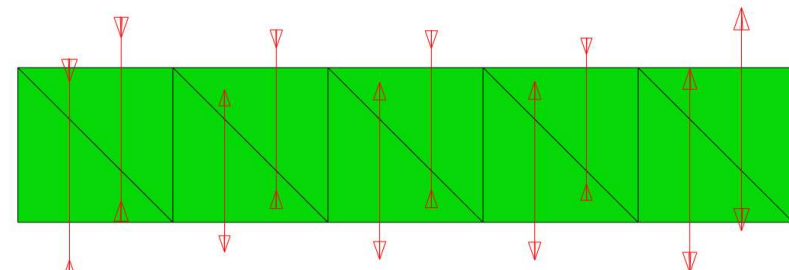
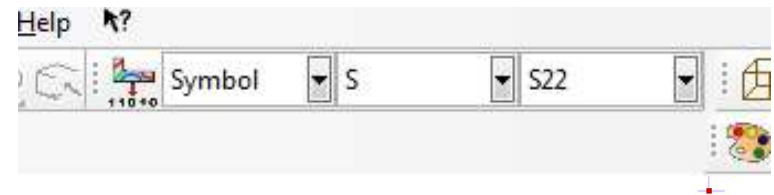
- Analysis, Create Job, Data Check, Submit
- Results
- Deformed plot, Stress plots
  - Field output, Mises, S11, S22, S12
  - Field output, U, U2
- Plot symbols
  - Plot stress direction
  - Make sure the "Field output tool box"



ODB: Job-1.odb Abaqus/Standard 6.9-1 Sat Oct 15 23:24:03 Eastern Daylight Time 2011

Step: Step-1  
Increment: 1; Step Time = 2.2200E-16  
Symbol Var: S, S11

S11



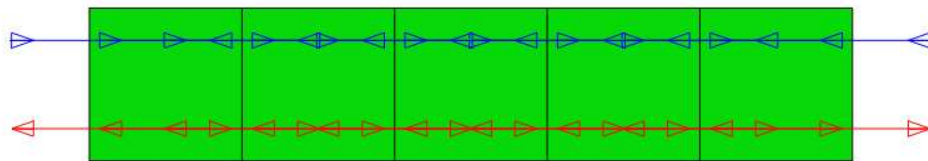
ODB: Job-1.odb Abaqus/Standard 6.9-1 Sat Oct 15 23:24:03 Eastern Daylight Time 2011

Step: Step-1  
Increment: 1; Step Time = 2.2200E-16  
Symbol Var: S, S22

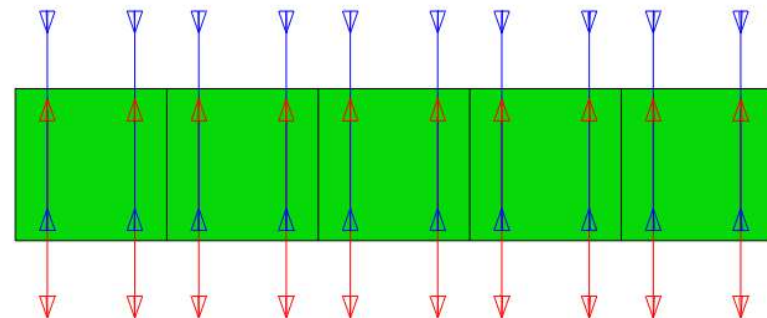
S22

# CANTILEVER BEAM (Q4)

- The same procedures of Parts to Loads as CST
- Mesh
  - Assign Mesh Controls, Quad (Quad only)
  - Element Type, Quad tap, uncheck "Reduced integration"
  - Global element size = 1
- Analysis, Create Job, Data Check, Submit
- Results
- Plot symbols



S11



S22



# Discussion

- Exact solution

$$(\sigma_{xx})_{\max} = 60\text{MPa} \quad \delta_{\max} = 7.5\text{mm}$$

- 10 Linear triangular elements

$$(\sigma_{xx})_{\max}^{\text{tri}} = 15.7\text{MPa} \quad \delta_{\max} = 1.794\text{mm}$$

- 5 Linear rectangular elements

$$(\sigma_{xx})_{\max}^{\text{rec}} = 44.4\text{MPa} \quad \delta_{\max} = 5.056\text{mm}$$

- 10 Quadratic triangular elements

$$(\sigma_{xx})_{\max}^{\text{tri}} = 60.0\text{MPa} \quad \delta_{\max} = 7.5\text{mm}$$

- 5 Quadratic rectangular elements

$$(\sigma_{xx})_{\max}^{\text{rec}} = 60.0\text{MPa} \quad \delta_{\max} = 7.5\text{mm}$$