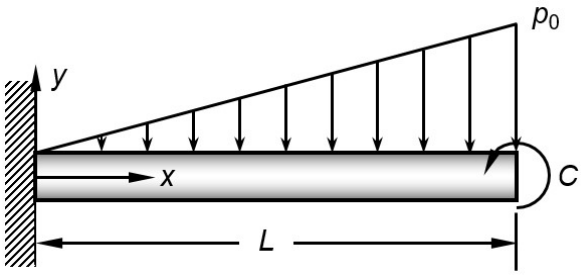
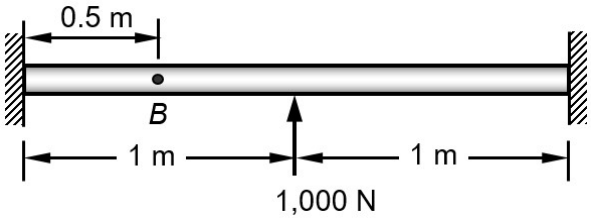
Homework #4

1. Use the Rayleigh-Ritz method to determine the deflection $v(x)$, bending moment $M(x)$, and shear force $V_y(x)$ for the beam shown in the figure. The bending moment and shear force are calculated from the deflection as: $M(x) = EI\mathrm{d}^2v / \mathrm{d}x^2$ and $V_y(x) = -EI\mathrm{d}^3v / \mathrm{d}x^3$. Assume the displacement as $v(x) = c_0 + c_1x + c_2x^2 + c_3x^3$, and EI = 2,000 N-m^2, L = 1 m, and p_0 = 200 N/m, and C = 100 N-m. Make sure the displacement boundary conditions are satisfied a priori.

   Hint: The potential energy of a couple is calculated as $V = -C\mathrm{d}v / \mathrm{d}x$, where the rotation is calculated at the point of application of the couple.



2. Use two equal-length beam elements to determine the deflection of the beam shown below. Estimate the deflection at point B, which is at 0.5 m from the left support. EI = 1000 N-m^2.



3. The frame shown in the figure is clamped at the left end and supported on a hinged roller at the right end. The radius of the circular cross section r = 0.04 m. An axial force P and a couple C act at the right end. Assume the following numerical values: L = 1 m, E = 80 GPa, P = 10,000 N, C = 1,000 Nm.

   (a) Use one element to determine the rotation θ at the right support.

   (b) What is the deflection of the beam at $x = L/2$?

   (c) What is the maximum tensile stress? Where does it occur?