# Finite Element Method

## A bridge between Mechanics of Materials and real-world applications


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# Point of Departure

Let's say that you have a cool idea of a new motorcycle design

So, you decide to build your own motorcycle

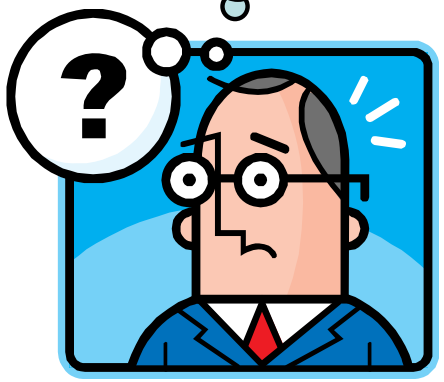
That means, you have to decide all parts, sizes, etc


# Design Trade-off

Need to remove material to have a good acceleration

Need to add material for safety

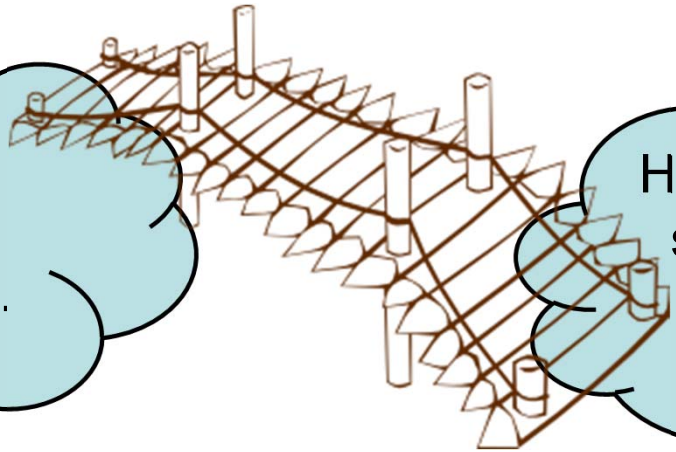
Where can I remove material? Where should I have to add material?


**Use Mechanics of Materials!!!**

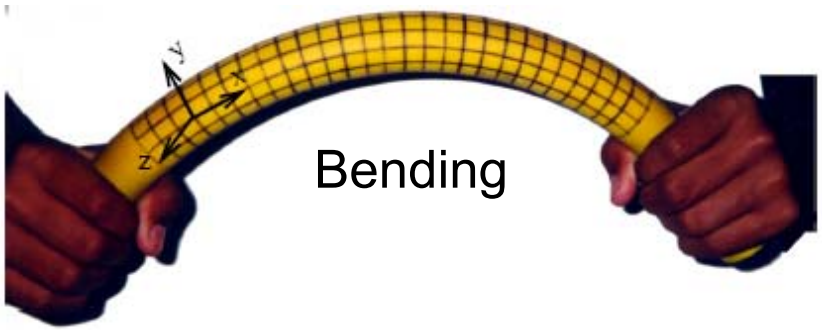
# Gap

We learned Mechanics of Materials, but …

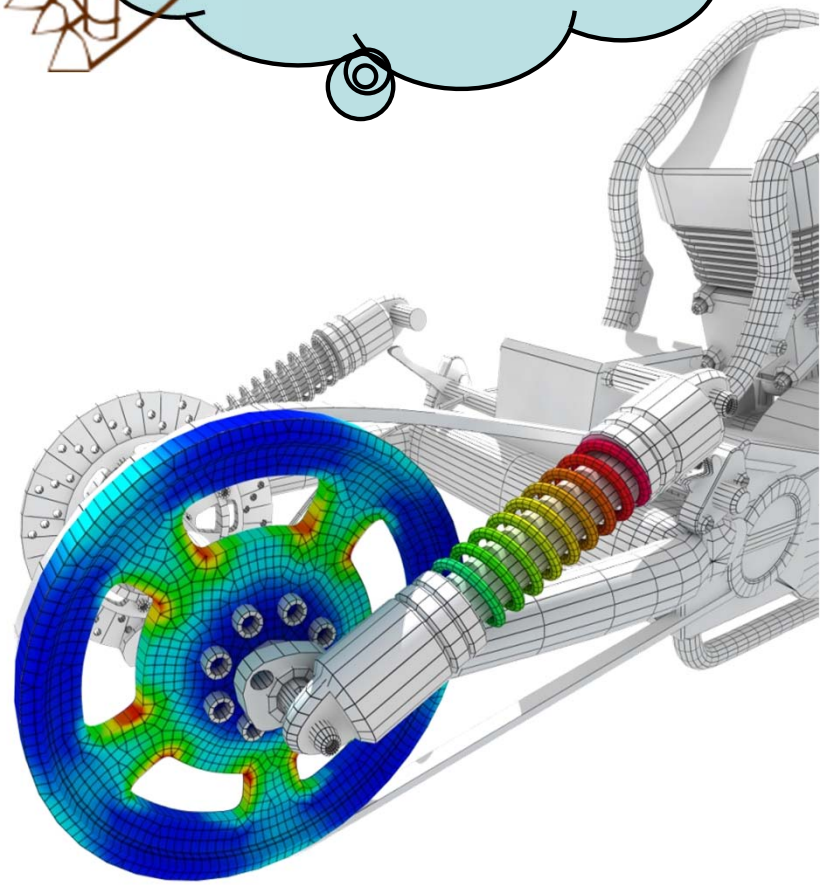


How can I use it to solve real-world applications?


Torsion




Bending



# Mechanics of Materials

Equilibrium at every point is governed by differential equation

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + b_x = 0$$

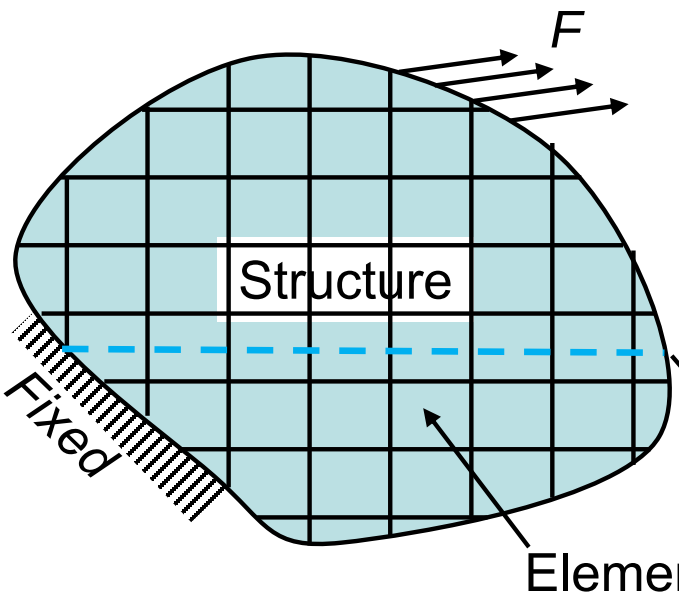
$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + b_y = 0$$

We know how to solve it in a simple domain

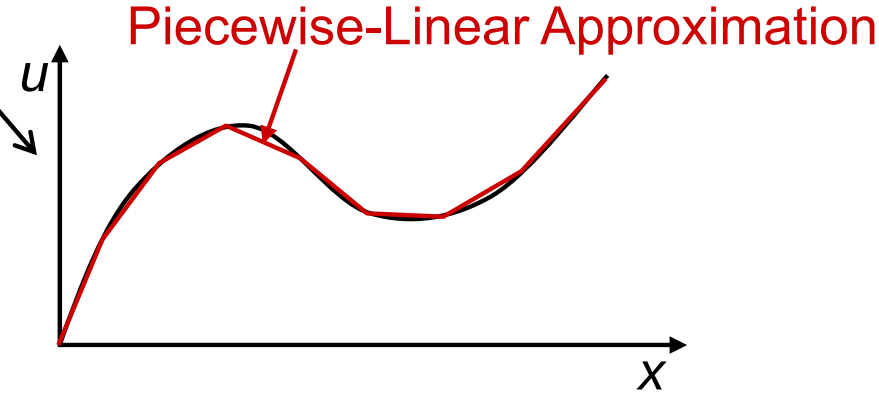

But, how can I solve for my motorcycle?

# Finite Element Method

- What is the finite element method (FEM)?
  - A technique for obtaining approximate solutions to boundary value problems.
  - Partition of the domain into a set of simple shapes (element)
  - Approximate the solution using piecewise polynomials within an element

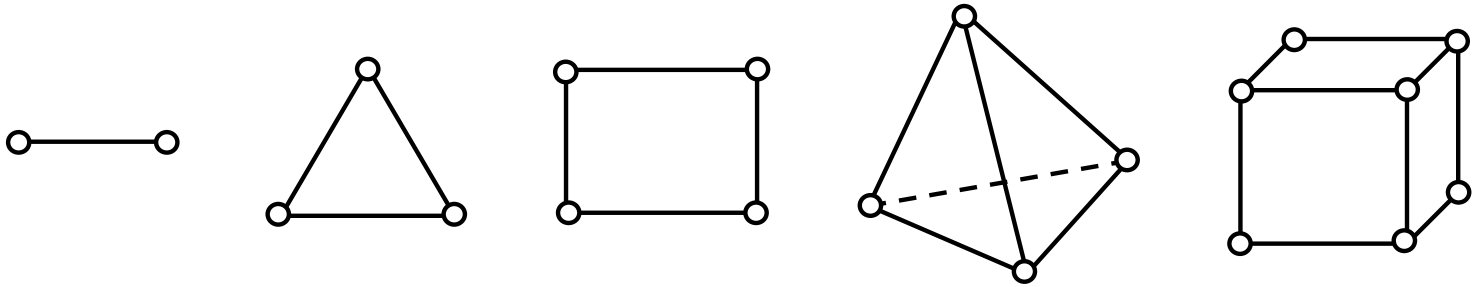



$$\left.\begin{aligned}\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+b_x=0\\ \frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}+b_y=0\end{aligned}\right\}$$

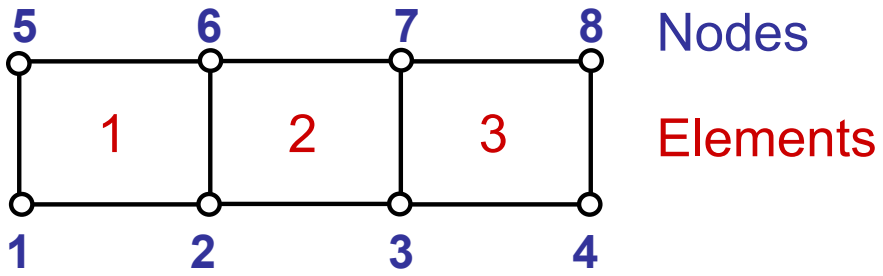


# What Are Elements?

- How to discretize the domain?
  - Using simple shapes (element)



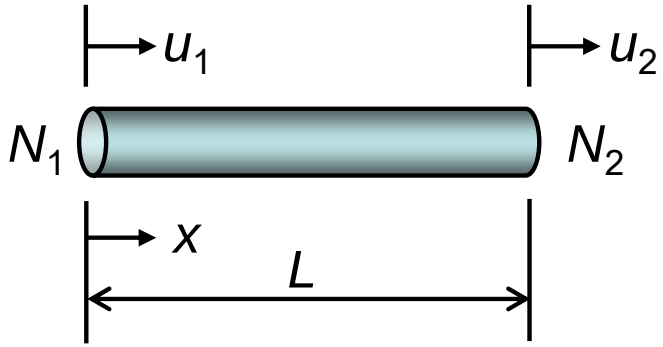
  - All elements are connected using “nodes”.




  - Solution at Element 1 is described using the values at Nodes 1, 2, 6, and 5 (Interpolation).
  - Elements 1 and 2 share the solution at Nodes 2 and 6.

# Interpolation

- Finite element analysis solves for Nodal Solutions.
  - All others can be calculated (or interpolated) from nodal solutions




  - Displacement within the element

$$u(x) = a + bx = u_1 + \frac{u_2 - u_1}{L}x = \frac{L - x}{L}u_1 + \frac{x}{L}u_2$$

Interpolation (Shape) Function

  - Strain of the element

$$\varepsilon(x) = \frac{\partial u}{\partial x} = -\frac{1}{L}u_1 + \frac{1}{L}u_2$$

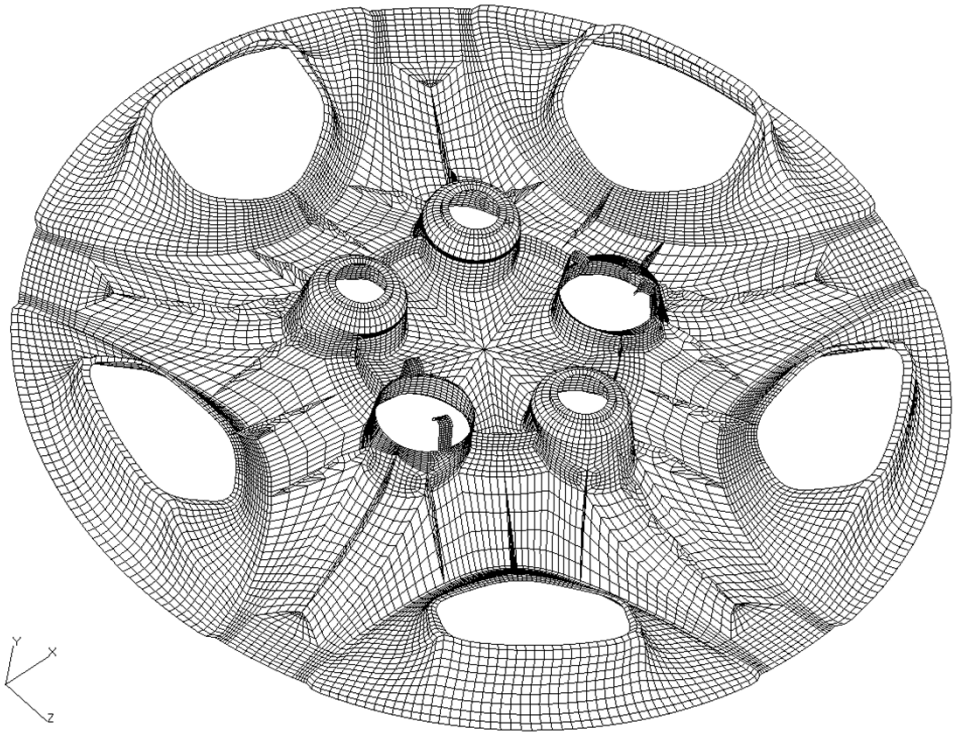
# System of Matrix Equations

- How to calculate nodal solutions?
  - Construct a huge simultaneous system of equations and solve for nodal solutions.
  - Different physical problems have different matrices and vectors.

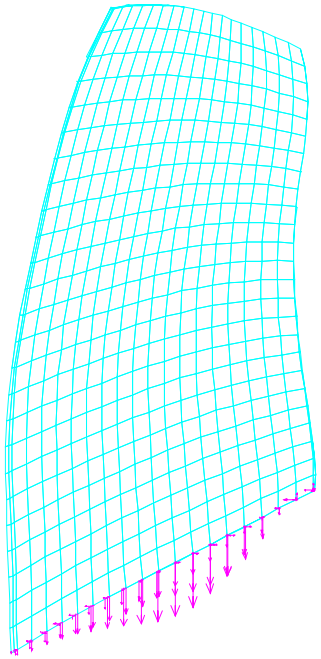
$$\begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1n} \\ K_{21} & K_{22} & \cdots & K_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ K_{n1} & K_{n2} & \cdots & K_{nn} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{Bmatrix}$$

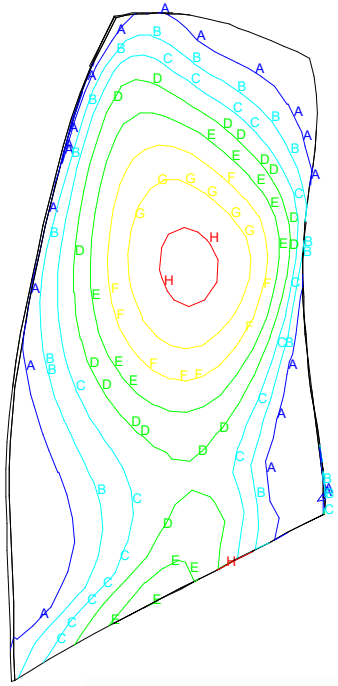
# Example: Finite Elements

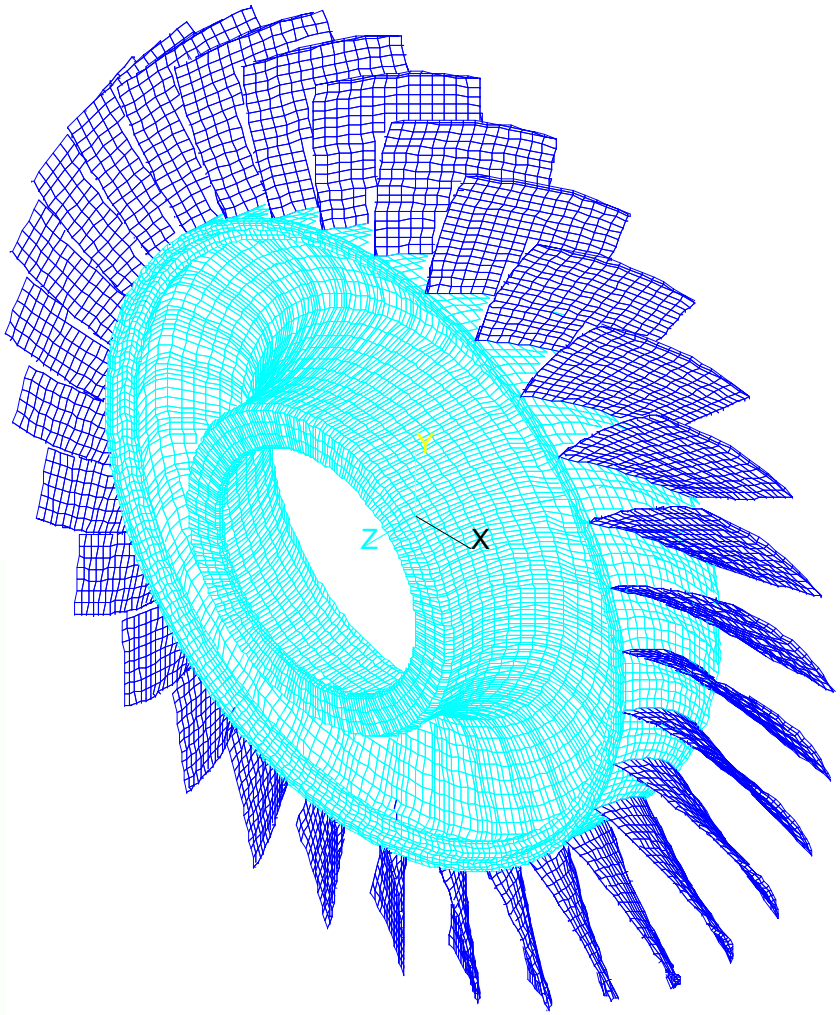
- Plastic Wheel Cover Model
  - 30,595 Nodes, 22,811 Elements
  - Matrix size is larger than 150,000×150,000.
  - MSC/PATRAN (Graphic user interface)

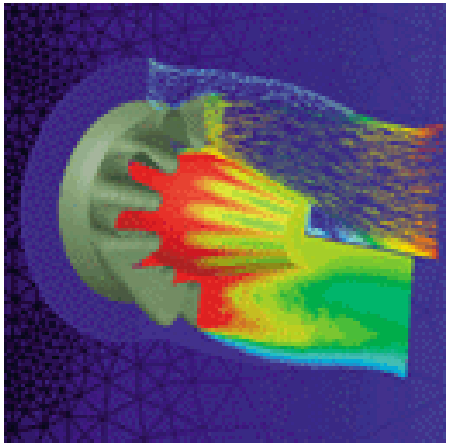


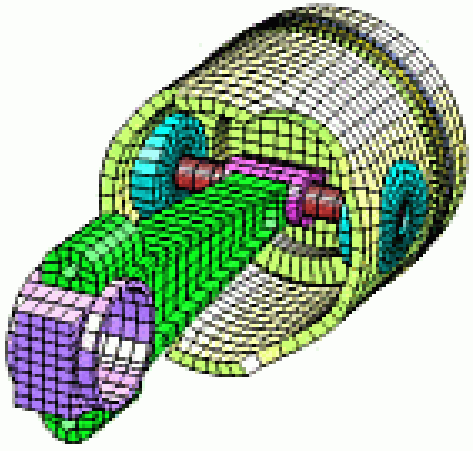
# Numerical Models of Engineering Components











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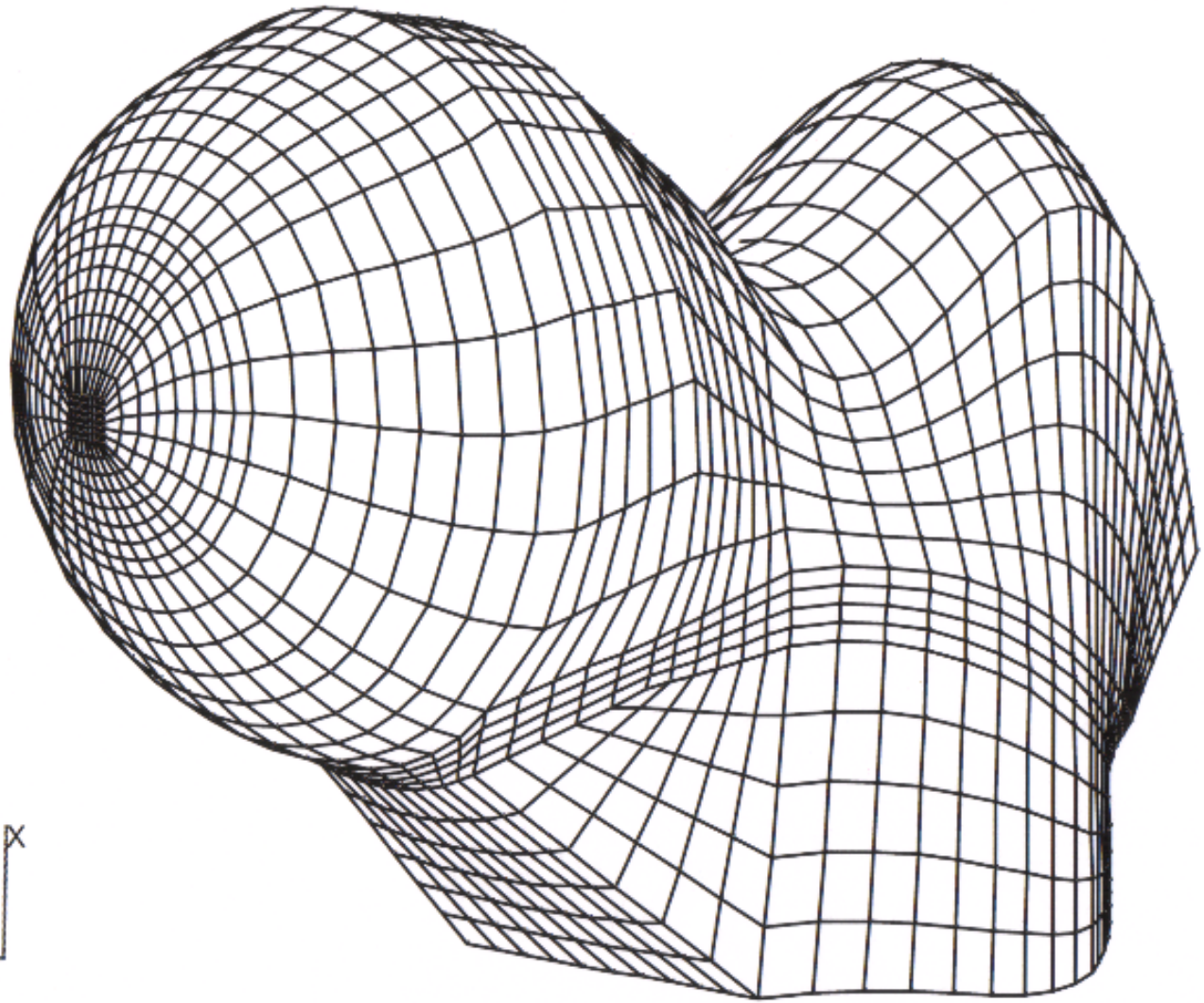
# Numerical Models of Engineering Components

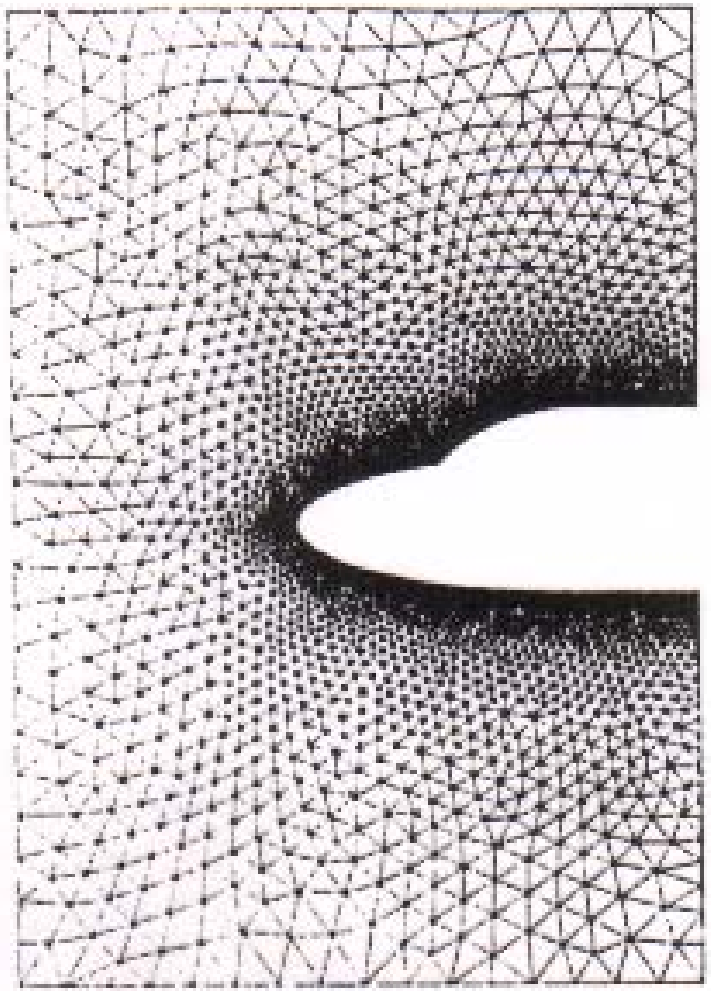


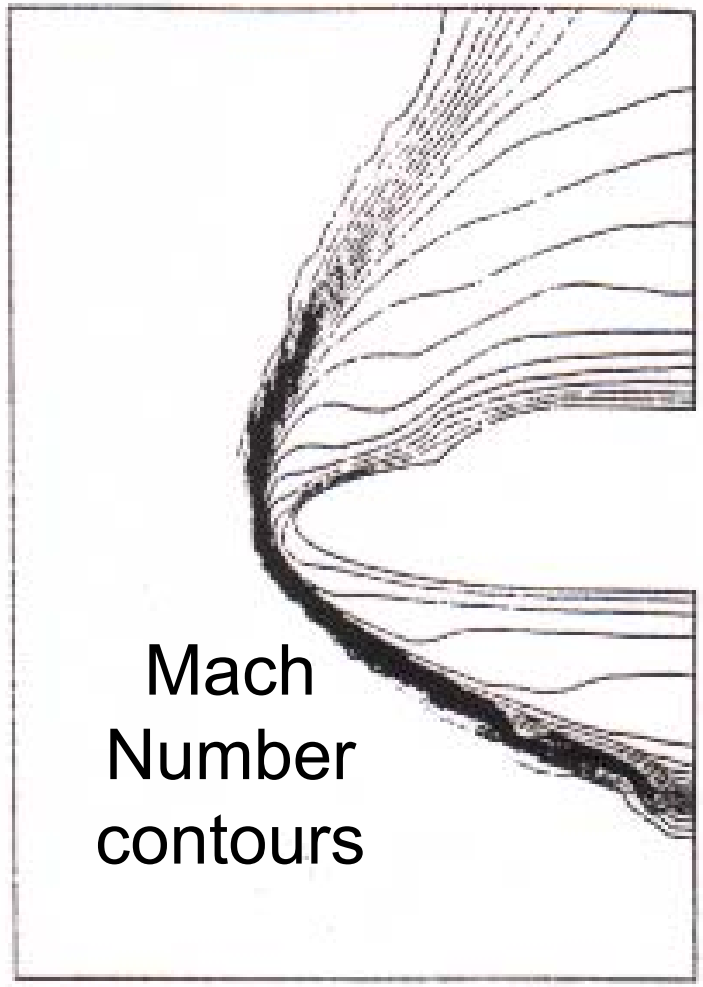

~21,000
unknowns

3-d finite element mesh for analysis: head of femur for 2-day old infant (after T. Ribble)

# Computational Fluid Dynamics

Initial Mesh

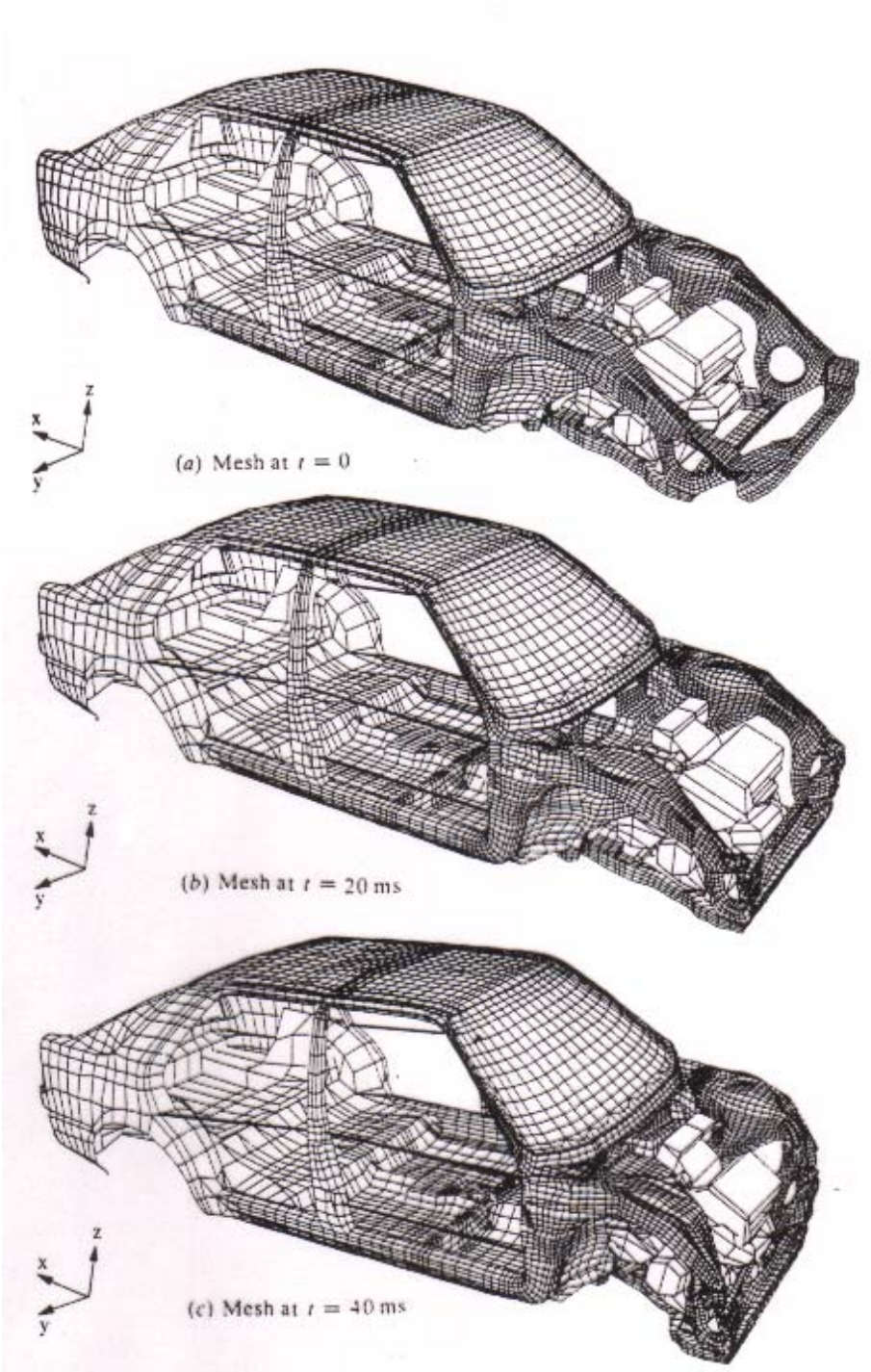





Viscous flow past 2-d simulation of the forebody of a shuttle at Mach 2.

(after Zienkiewicz and Taylor, 1991).

# Crashworthiness Analysis



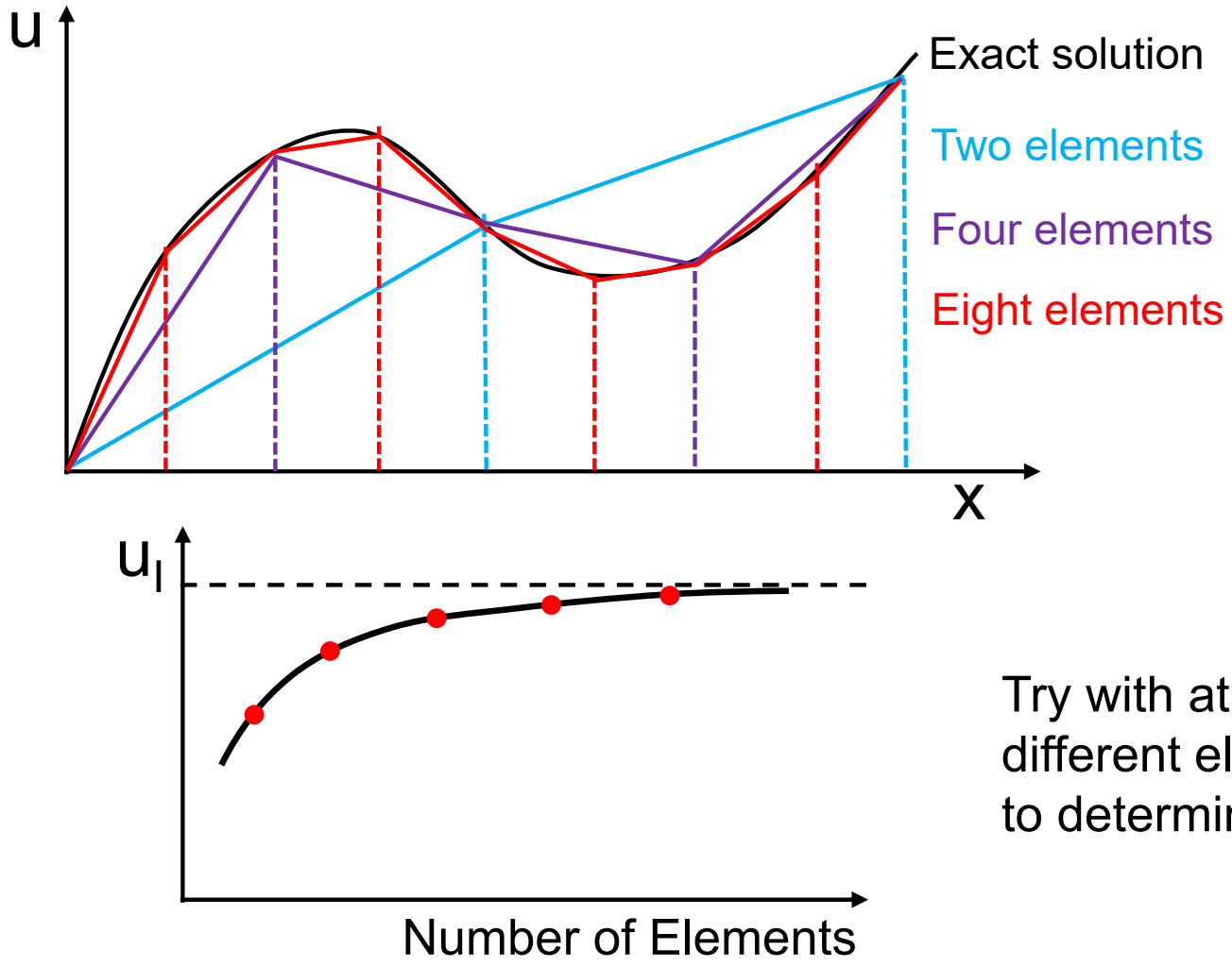

Crash of a
SAAB 9000

17h CPU time on
CRAY x-MP/48

(after J. Hallquist)

# Convergence Study

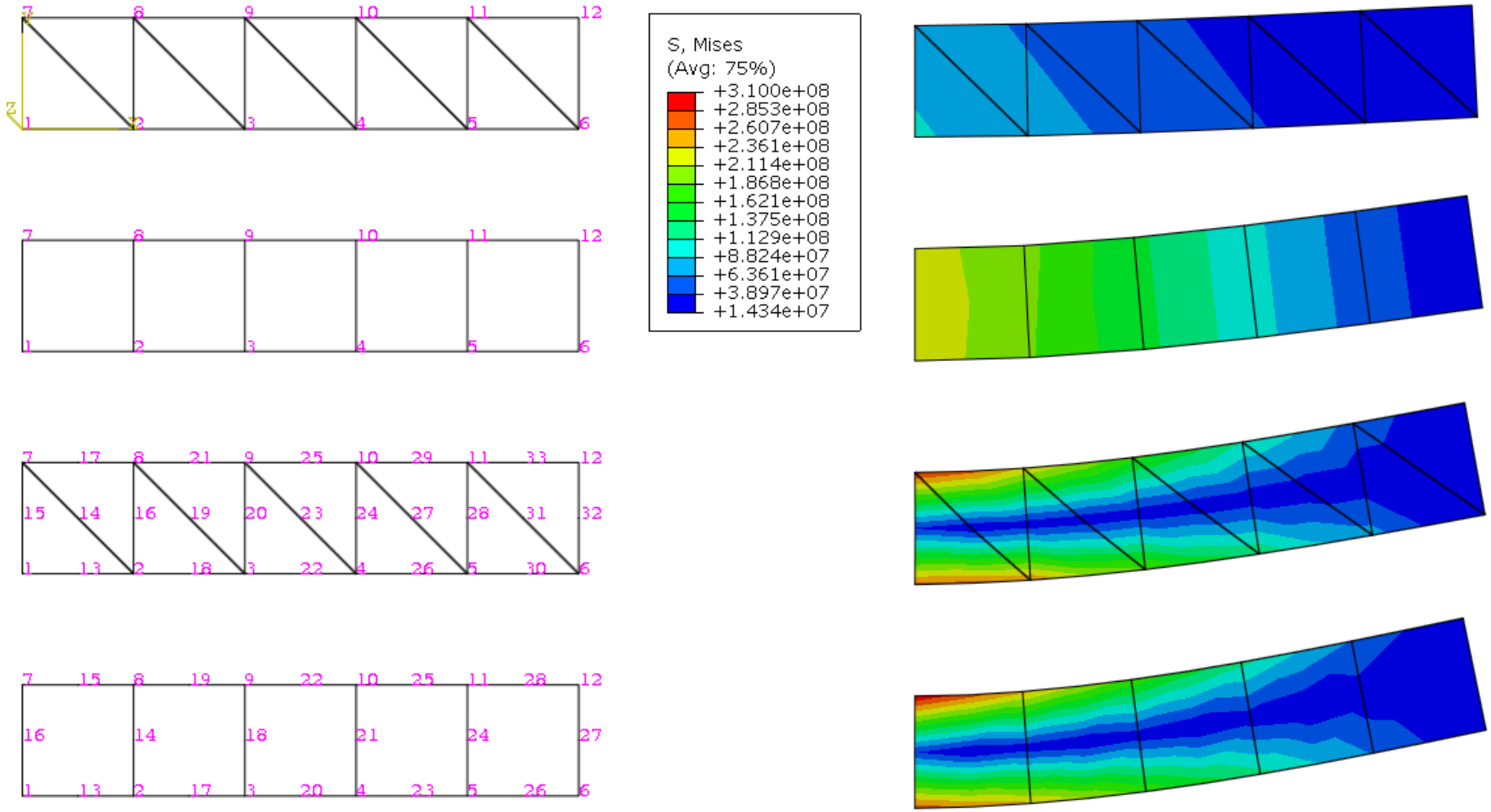
- How do you know the FEM solution is accurate?
- Convergence: the finite element solution converges to the exact solution as the size of elements decreases



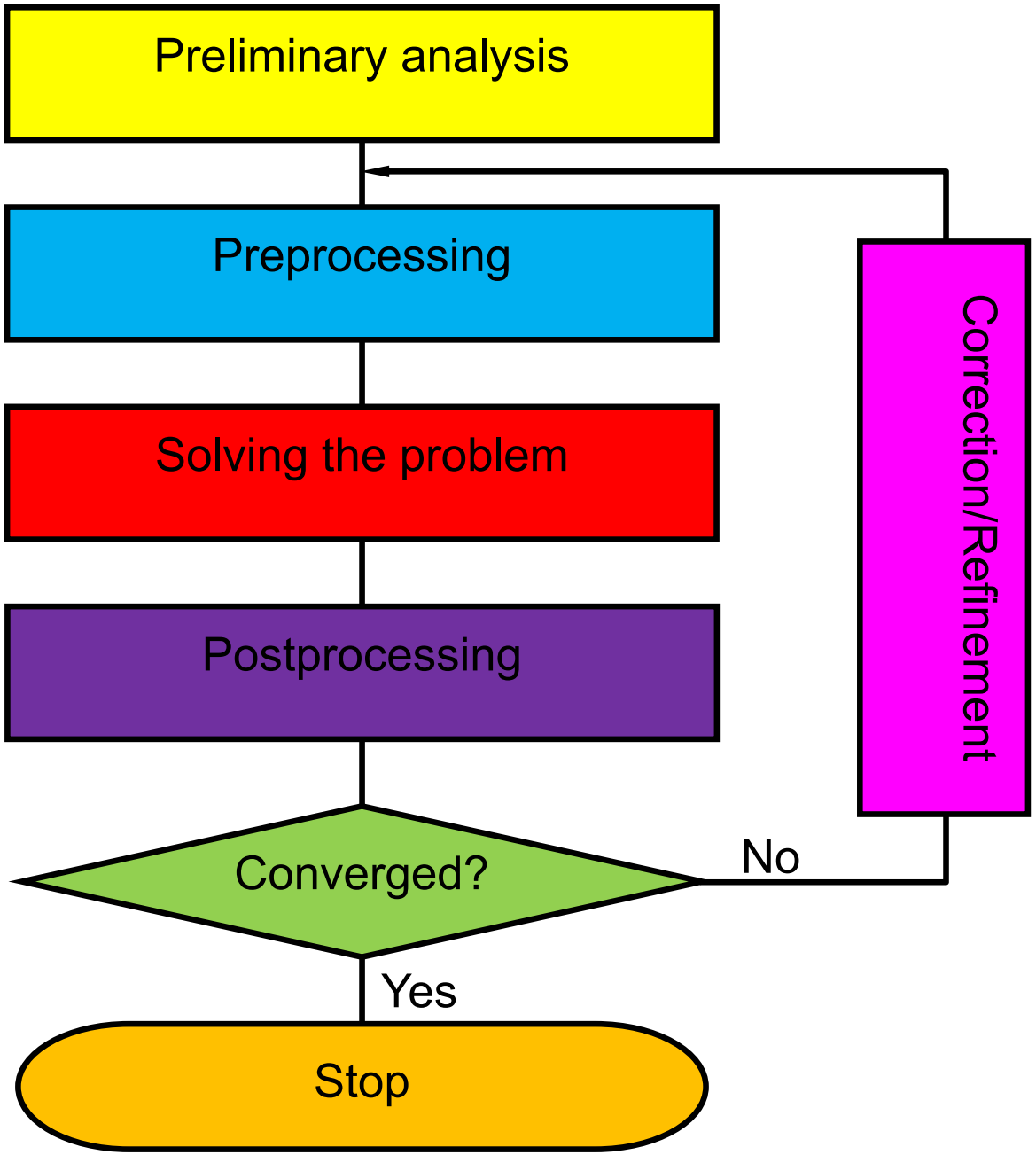

Try with at least three different element sizes to determine convergence

# Element Selection

- What element should I have to use?
  - Element is mathematical representation
  - Different elements behave differently

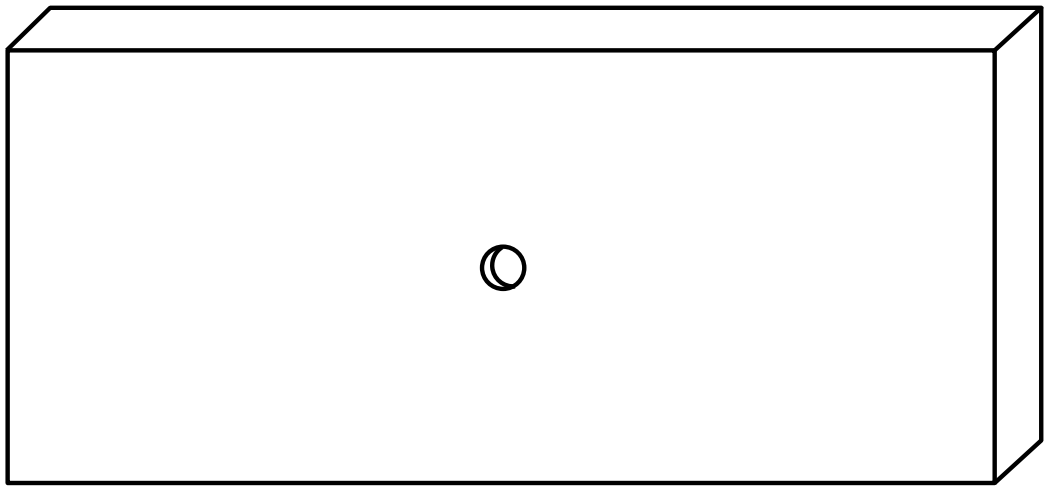


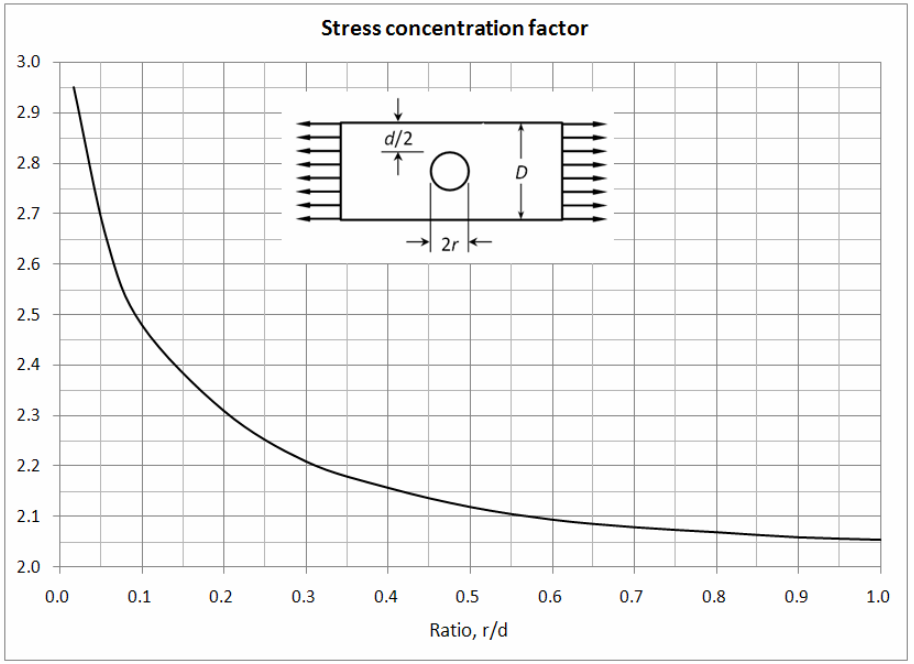
# Finite Element Procedure



# Modeling Issues

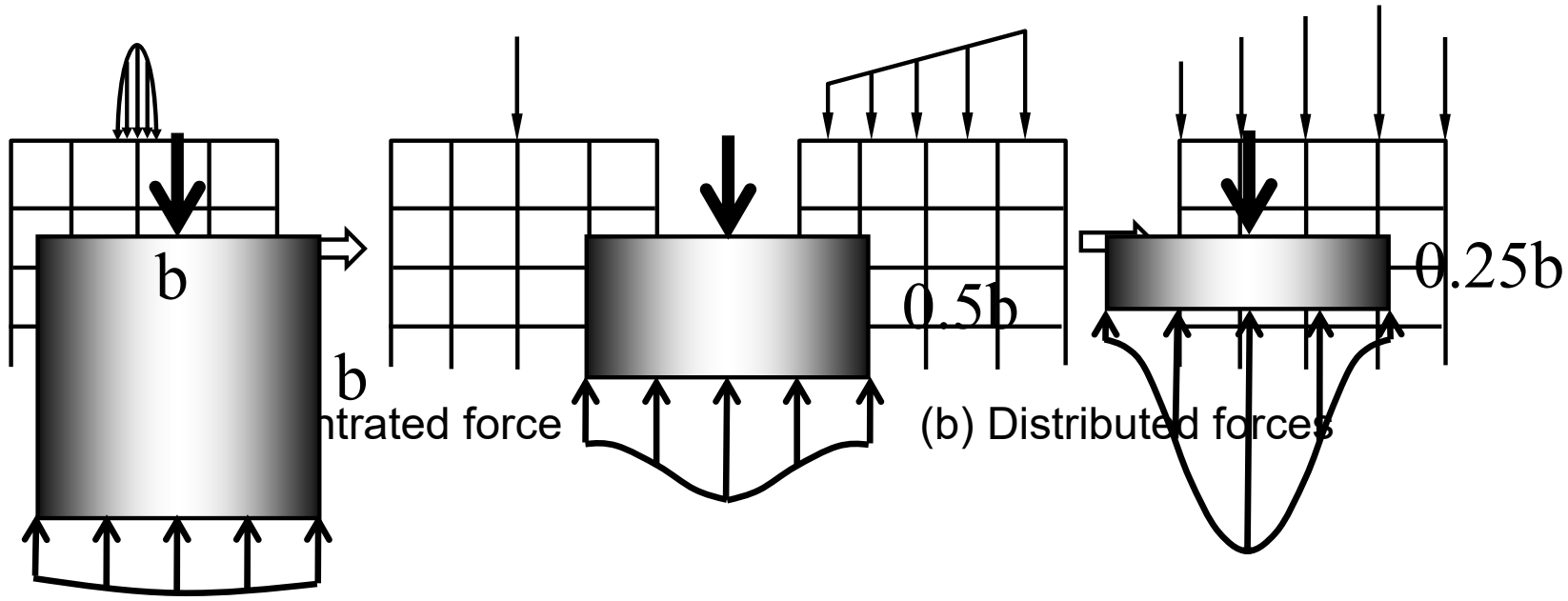
- Common mistake: FE model is not a replication of CAD geometry
- Model: Mathematically identical to a purpose
- Simplification: delete unimportant small features
  separate consideration of small holes





# Boundary Conditions

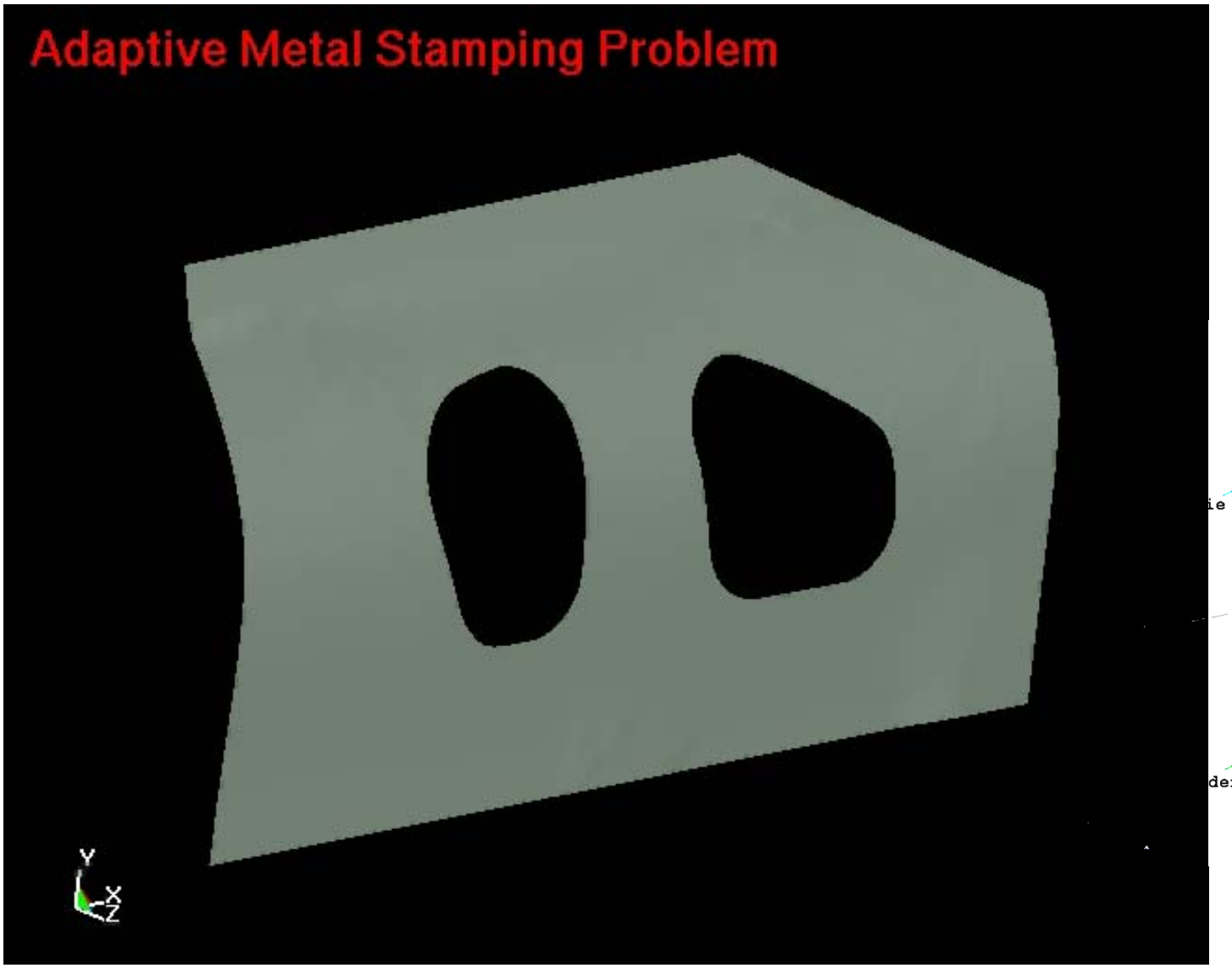
- Error in boundary conditions will not disappear no matter how much you refine the model!!
- Most assumptions are often made in BC
- Need to be careful in interpreting results near the boundary

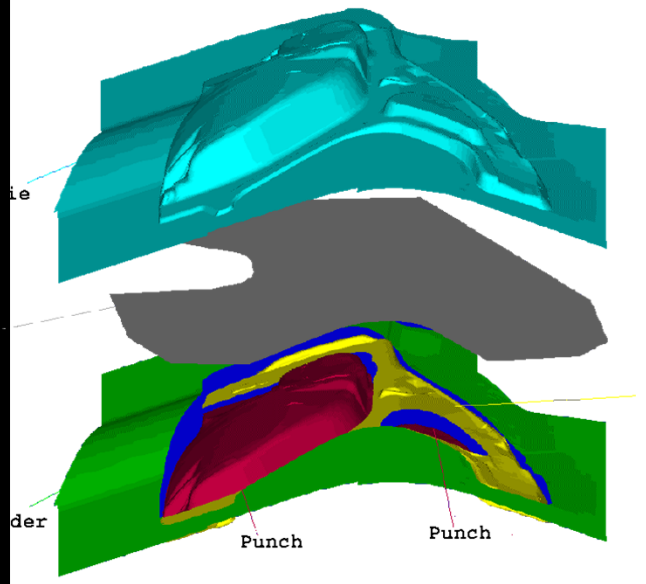



$\sigma_{min} = 0.973\sigma_{ave}$ $\sigma_{min} = 0.668\sigma_{ave}$ $\sigma_{min} = 0.198\sigma_{ave}$

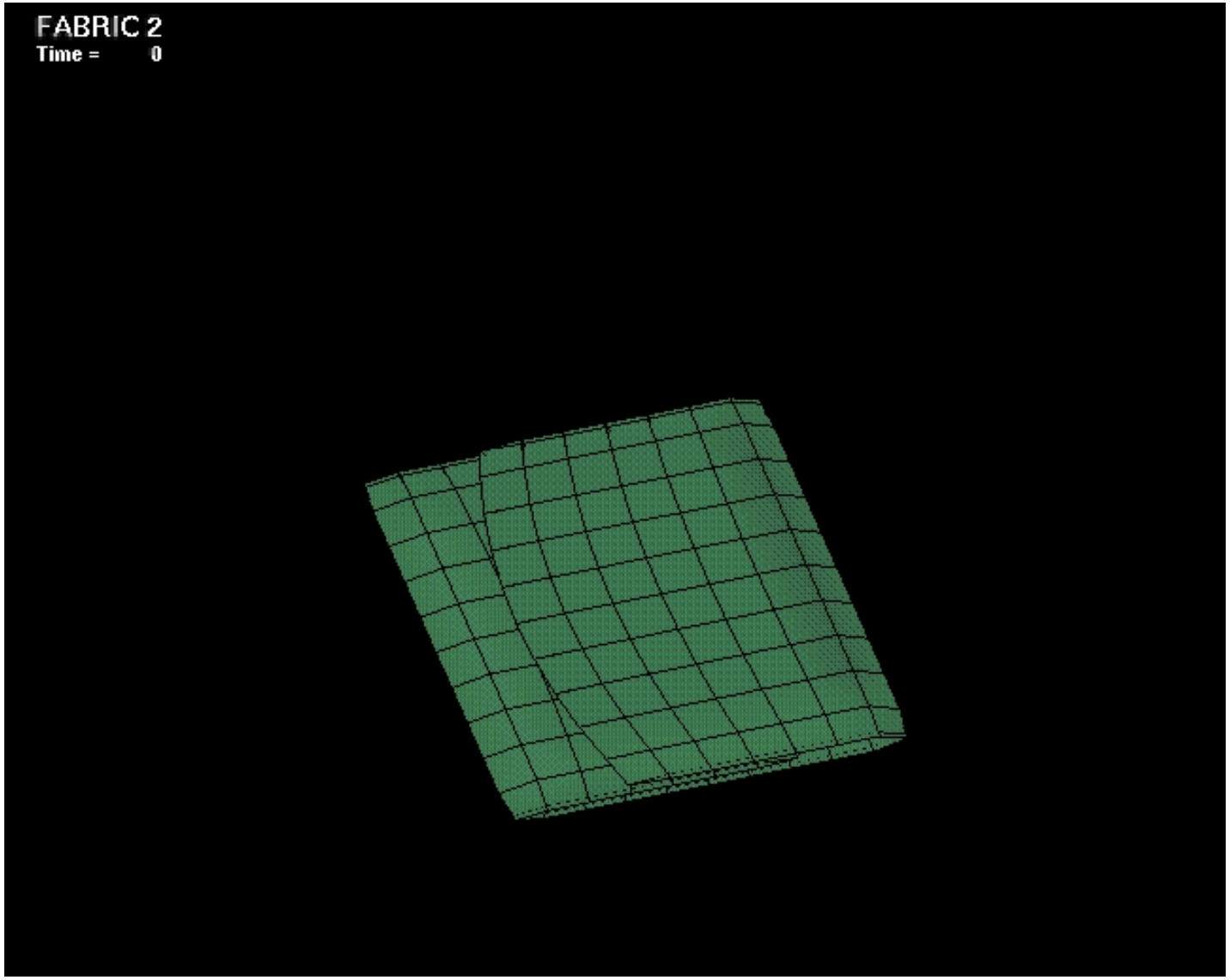
$\sigma_{max} = 1.027\sigma_{ave}$ $\sigma_{max} = 1.387\sigma_{ave}$ $\sigma_{max} = 2.575\sigma_{ave}$

# Example: Automotive Door Panel Stamping

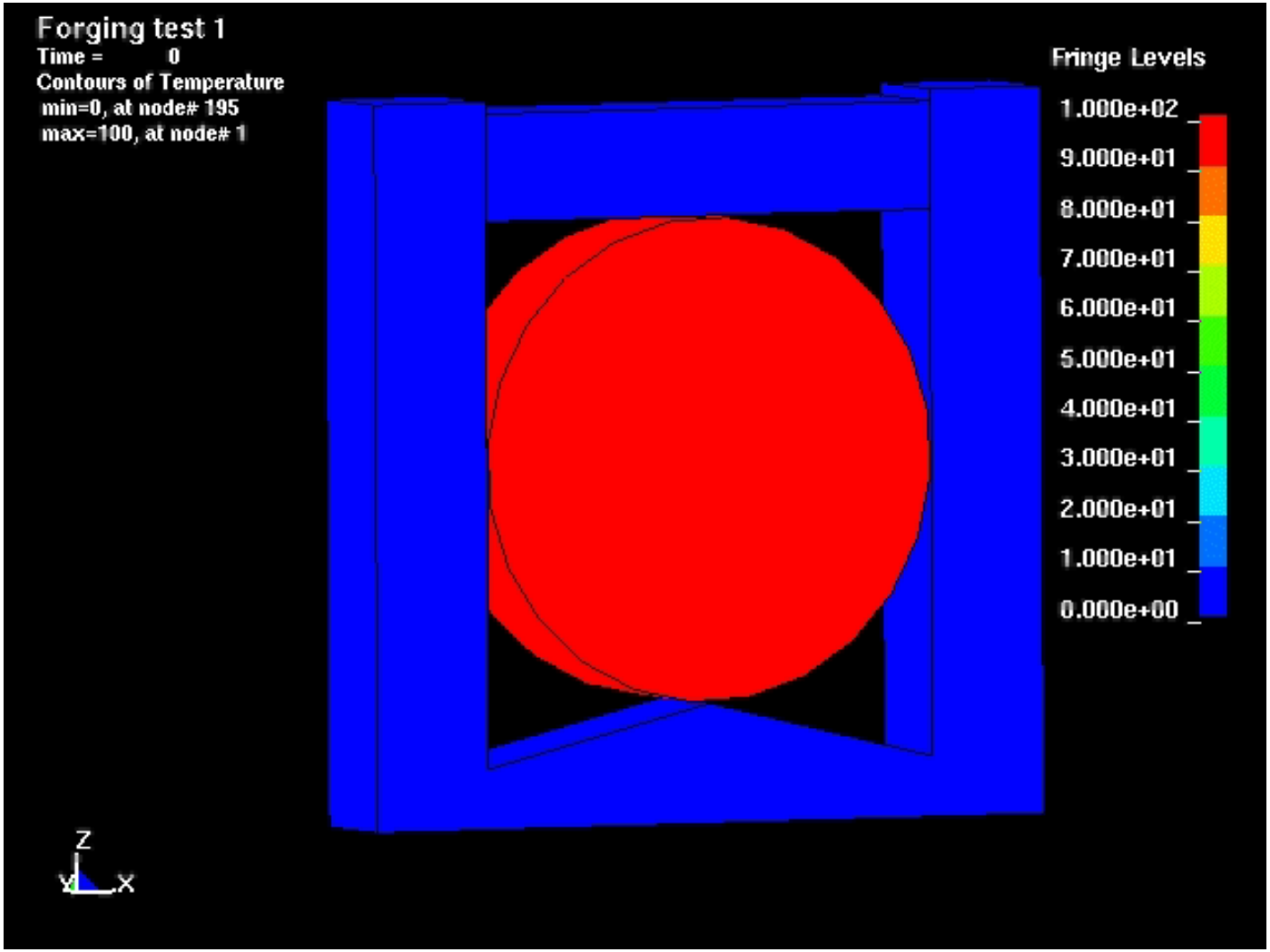


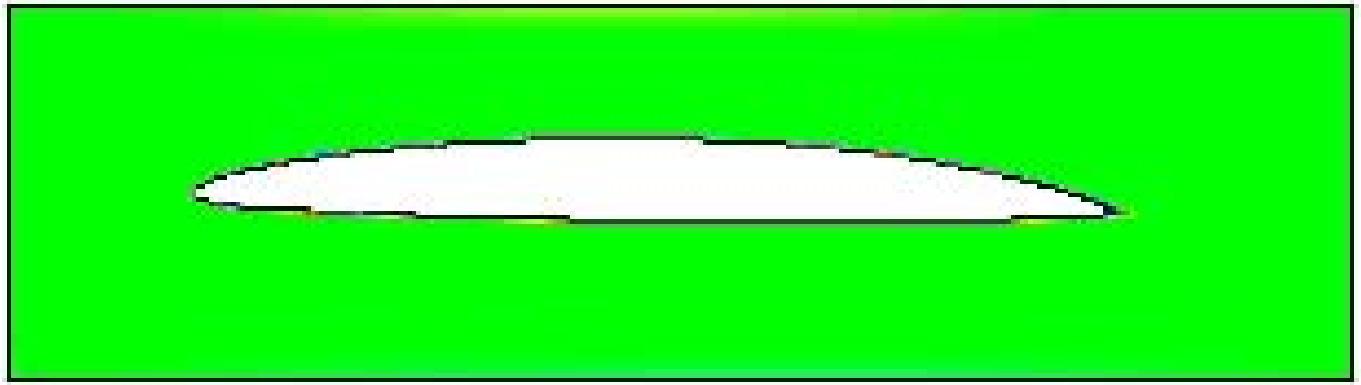
# Example: Airbag Deployment



# Forging


Forging test 1
Time = 0
Contours of Temperature
min=0, at node# 195
max=100, at node# 1
Fringe Levels
1.000e+02
9.000e+01
8.000e+01
7.000e+01
6.000e+01
5.000e+01
4.000e+01
3.000e+01
2.000e+01
1.000e+01
0.000e+00
Z
Y
X

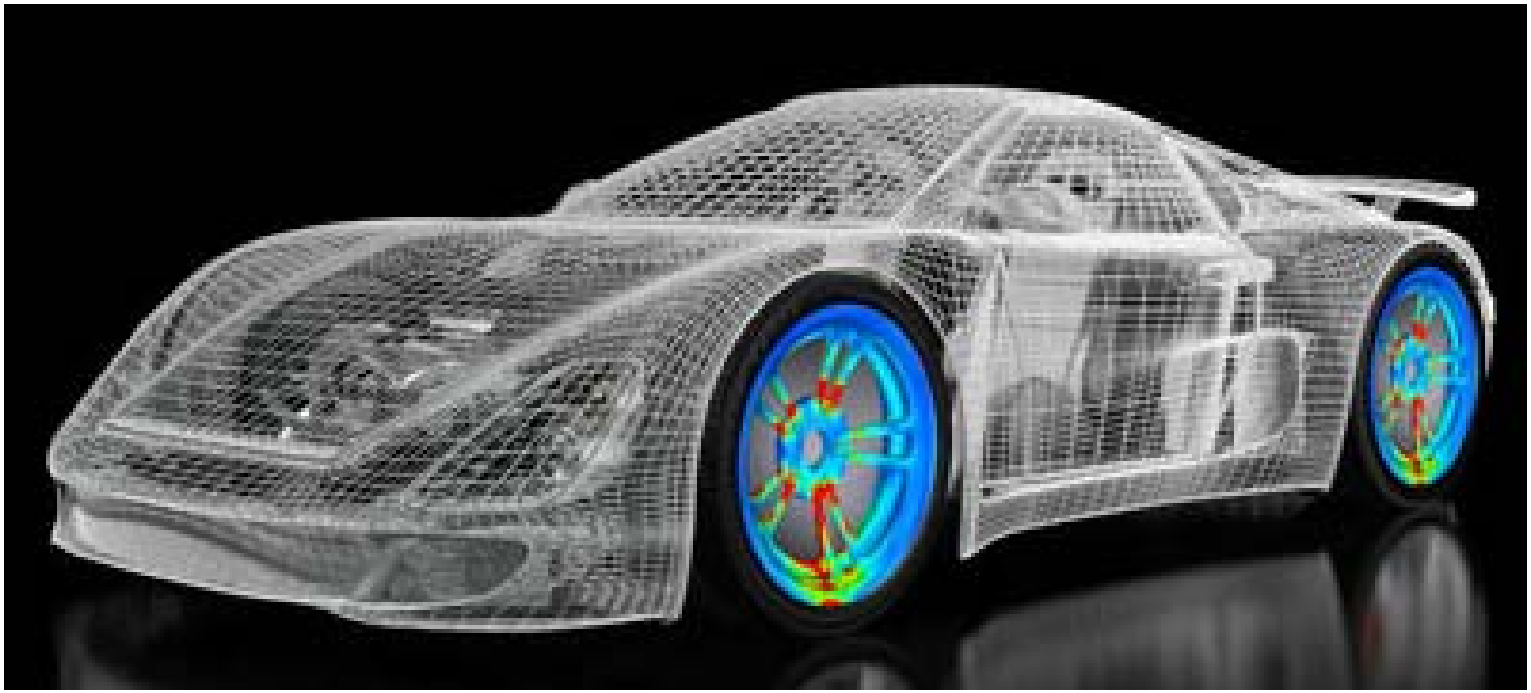
# Example: Vorticity



# Things to Remember

- Finite element method does not solve a problem, but it helps YOU to solve the problem
- It helps you to understand the mechanical system that you are working on
- Garbage inputs, garbage outputs
- Try to be an engineer, not a technician

# QUESTIONS?



# YouTube Videos

- Two vehicle impact NCAC FEA FEM
  - http://youtu.be/hrfcROMz2II
- Downmilling
  - http://youtu.be/pYCOkIVLA_c
- Rolling
  - http://youtu.be/E1d4WKkbtdY