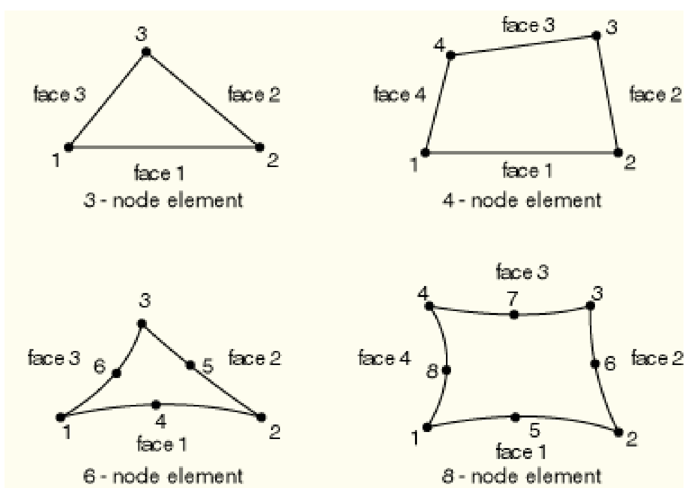
NONLINEAR ELASTIC ANALYSIS USING ABAQUS

2D Solid (Continuum) Elements

- Plane strain
  - CPE3 3-node linear
  - CPE4 4-node bilinear
  - CPE6 6-node quadratic
  - CPE8 8-node biquadratic
- Plane stress
  - CPS3 3-node linear
  - CPS4 4-node bilinear
  - CPS6 6-node quadratic
  - CPS8 8-node biquadratic
- Distributed body forces (*DLOAD)
- Surface forces (*DSLOAD)



Stress & Strain Measures in ABAQUS

- ABAQUS uses the updated Lagrangian formulation
- Stress measure
  - ABAQUS always calculates **Cauchy (true) stress**
- Total (integrated) strain
  - Default strain output (E). Accumulation of incremental strains

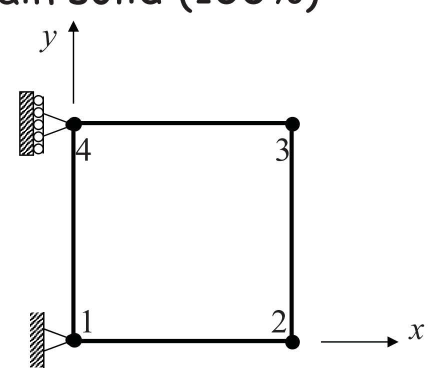
$$\varepsilon^{n+1} = \Delta\mathbf{R}\cdot\varepsilon^{n}\cdot\Delta\mathbf{R}^{T} + \Delta\varepsilon$$

- Nominal strain (NE) $\varepsilon^{N} = \mathbf{V} - \mathbf{1} = \sum_{i=1}^{3}(\lambda_i - 1)\mathbf{e}_i \otimes \mathbf{e}_i$
- Logarithmic strain (LE) $\varepsilon^{L} = \ln\mathbf{V} = \sum_{i=1}^{3}\ln\lambda_i \mathbf{e}_i \otimes \mathbf{e}_i$
- Green-Lagrange strain $\varepsilon^{G} = \frac{1}{2}(\mathbf{F}^{T}\mathbf{F} - \mathbf{1})$

Uniaxial Extension

- X-directional extension of a plane strain solid (100%)
- Elastic incompressible material (E = 200 GPa, ν = 0.499)
- Mapping relation (λ_1 = 2, λ_2 = 0.5)

$$x_1 = 2X_1 \qquad x_2 = 0.5X_2 \qquad F = \begin{bmatrix} 2 & 0 \\ 0 & .5 \end{bmatrix}$$



- Nominal strain

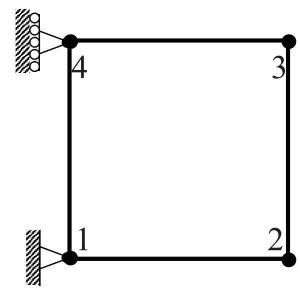
$$\varepsilon^{N} = \mathbf{V} - \mathbf{1} = \sum_{i=1}^{3}(\lambda_i - 1)\mathbf{e}_i \otimes \mathbf{e}_i = \begin{bmatrix} 1 & 0 \\ 0 & -.5 \end{bmatrix}$$

- Logarithmic strain

$$\varepsilon^{L} = \ln\mathbf{V} = \sum_{i=1}^{3}\ln\lambda_i \mathbf{e}_i \otimes \mathbf{e}_i = \begin{bmatrix} \ln 2 & 0 \\ 0 & -\ln 2 \end{bmatrix} = \begin{bmatrix} .6931 & 0 \\ 0 & -.6931 \end{bmatrix}$$

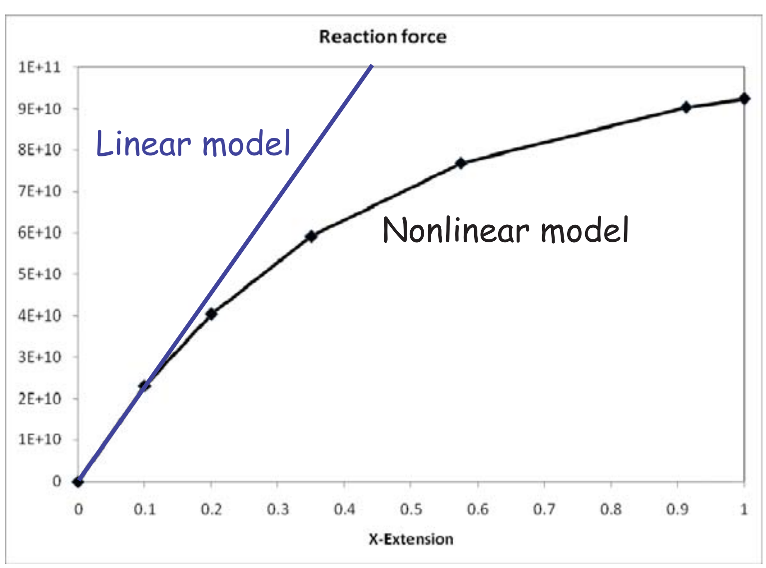
Uniaxial Extension cont.

```
*HEADING
2D plane strain element large
    deformation
SI units (kg, m, s, N)
*PREPRINT, ECHO=YES, MODEL=YES,
    HISTORY=YES
**
** Model definition
*NODE, NSET=NALL
1, 0.,  0., 0.
2, 1.,  0., 0.
3, 1.,  1., 0.
4, 0.,  1., 0.
*ELEMENT, TYPE=CPE4,
    ELSET=SQUARE
1, 1, 2, 3, 4
*SOLID SECTION, ELSET=SQUARE,
    MATERIAL=STEEL
1.0,
*MATERIAL, NAME=STEEL
*ELASTIC
200.E9, 0.499
**
** History data
*STEP, NLGEOM=YES, INC=150
Extension 100%
*STATIC
0.1, 1.0, 0.0001, 0.5
*BOUNDARY
1, 1, 2
2, 2,
2, 1, 1, 1.0
3, 1, 1, 1.0
4, 1,
*NODE PRINT
U,
RF,
*EL PRINT
LE,
NE,
S,
*END STEP
```



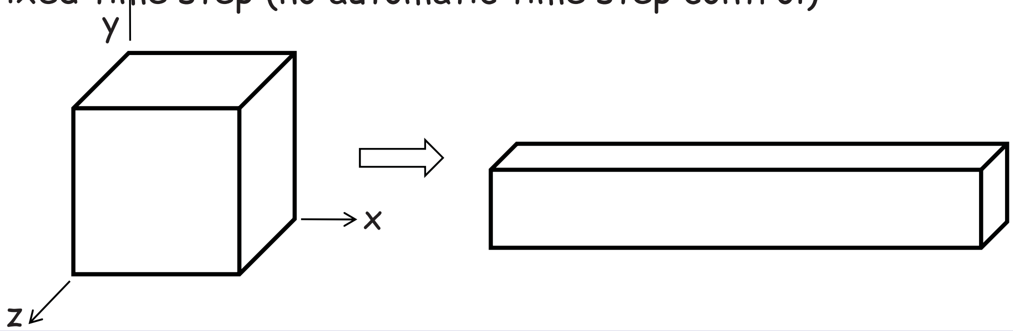
Uniaxial Extension cont

- Show squareTension.dat
- Show postprocessing
- Force-displacement relation



Hyperelastic Material Analysis Using ABAQUS

- *ELEMENT,TYPE=C3D8RH,ELSET=ONE
  - 8-node linear brick, reduced integration with hourglass control, hybrid with constant pressure
- *MATERIAL,NAME=MOONEY
  *HYPERELASTIC, MOONEY-RIVLIN
  80., 20.,
  - Mooney-Rivlin material with A_{10} = 80 and A_{01} = 20
- *STATIC,DIRECT
  - Fixed time step (no automatic time step control)



Hyperelastic Material Analysis Using ABAQUS

```
*HEADING
 - Incompressible hyperelasticity (Mooney-
   Rivlin) Uniaxial tension
*NODE,NSET=ALL
1,
2,1.
3,1.,1.,
4,0.,1.,
5,0.,0.,1.
6,1.,0.,1.
7,1.,1.,1.
8,0.,1.,1.
*NSET,NSET=FACE1
1,2,3,4
*NSET,NSET=FACE3
1,2,5,6
*NSET,NSET=FACE4
2,3,6,7
*NSET,NSET=FACE6
4,1,8,5
*ELEMENT,TYPE=C3D8RH,ELSET=ONE
1,1,2,3,4,5,6,7,8
*SOLID SECTION, ELSET=ONE,
   MATERIAL= MOONEY
```

```
*MATERIAL,NAME=MOONEY
*HYPERELASTIC, MOONEY-RIVLIN
80., 20.,
*STEP,NLGEOM,INC=20
UNIAXIAL TENSION
*STATIC,DIRECT
1.,20.
*BOUNDARY,OP=NEW
FACE1,3
FACE3,2
FACE6,1
FACE4,1,1,5.
*EL PRINT,F=1
S,
E,
*NODE PRINT,F=1
U,RF
*OUTPUT,FIELD,FREQ=1
*ELEMENT OUTPUT
S,E
*OUTPUT,FIELD,FREQ=1
*NODE OUTPUT
U,RF
*END STEP
```

Hyperelastic Material Analysis Using ABAQUS

- Analytical solution procedure
  - Gradually increase the principal stretch λ from 1 to 6
  - Deformation gradient

$$\mathbf{F} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & 1/\sqrt{\lambda} & 0 \\ 0 & 0 & 1/\sqrt{\lambda} \end{bmatrix}$$

  - Calculate $J_{1,E}$ and $J_{2,E}$
  - Calculate 2nd P-K stress

$$\mathbf{S} = A_{10}\mathbf{J}_{1,E} + A_{01}\mathbf{J}_{2,E}$$

  - Calculate Cauchy stress

$$\sigma = \frac{1}{J}\mathbf{F} \cdot \mathbf{S} \cdot \mathbf{F}^T$$

  - Remove the hydrostatic component of stress

$$\sigma_{11} = \sigma_{11} - \sigma_{22}$$

Hyperelastic Material Analysis Using ABAQUS

- Comparison with analytical stress vs. numerical stress